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Shear stress distribution in haunched beam models

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SHEAR STRESS DISTRIBUTION

IN

HAUNCHED BEAM MODELS

BY

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A

THESIS

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ABSTRACT

The purpose of this thesis has been to give the distribution of shear stress inside a haunch. A haunch is generally provided to relieve high stress concentrations at the face of the column. However the nature of shear stress distribution inside the haunch is not known.

This stress distribution was determined from a photoelastic study on a CR-39 model. It is noted that the haunch reduces the stress concentration at the face of the column but at the same time somewhere near the point where the haunch meets the beam there is a high stress concentration.

Shear stress changes direction within the haunch and a large part of the shear is taken up by the material which is in the lower half of the haunch.

It is also noted that a haunch with an angle greater than 45 degrees with the horizontal is better than one with an angle less than 45 degrees.

I. INTRODUCTION

A beam and column junction is generally haunched for a two-fold purpose.

1) To relieve high stress concentration at the junction

2) To improve the appearance.

To a structural designer the first of the aforementioned is the only important purpose because in structural design it is assumed that an increase in depth will always take care of deformations due to shear and stress-concentration and thus the only effect of a haunch is to increase negative bending moment at the support and decrease positive bending moment near the center of the span. The fact still remains however that the haunch has relatively sharp corners at the junction of the beam and column and high stress concentration is expected at or near these points.

The purpose of this study is therefore to determine the shear stress distribution inside the haunch and determine the section at which it is maximum. This can be of great help to a designer because it gives a general distribution of shear stresses near the point of stress concentration.

Here the problem is approached through experimental analysis and the following methods can be used for such an analysis.

(1) Electrical Strain Gauges

(2) Grid method

(3) Brittle coating methods

(4) Photo-elastic method

The solution of this problem is to be tried by the photo-elastic method because:

(1) Photo-elastic models are made with very little material and can be made in relatively shorter time.

(2) The calculations of shear stresses is comparetively more simple than in the first three methods.

(3) After the preparation of models no additional processing is done and hence the results depend entirely on the characteristics of the model.

The next step is the selection of the material. Columbia Resin-39 was used for the preparation of models in this analysis. It has a low value of fringe stress coefficient so that a large number of fringes is produced for a given amount of stress and better accuracy is obtained in calculations. It also has a high elastic limit so that more fringes can be obtained within the elastic range. It has a good machinability so that cutting and grinding is performed easily and both the surfaces are highly polished so that it gives a good image of fringes and isoclinics. It is also comparatively free from time edge effect.

The disadvantage is the brittleness of this material, but this was avoided by careful handling while machining the model.

II. REVIEW OF LITERATURE

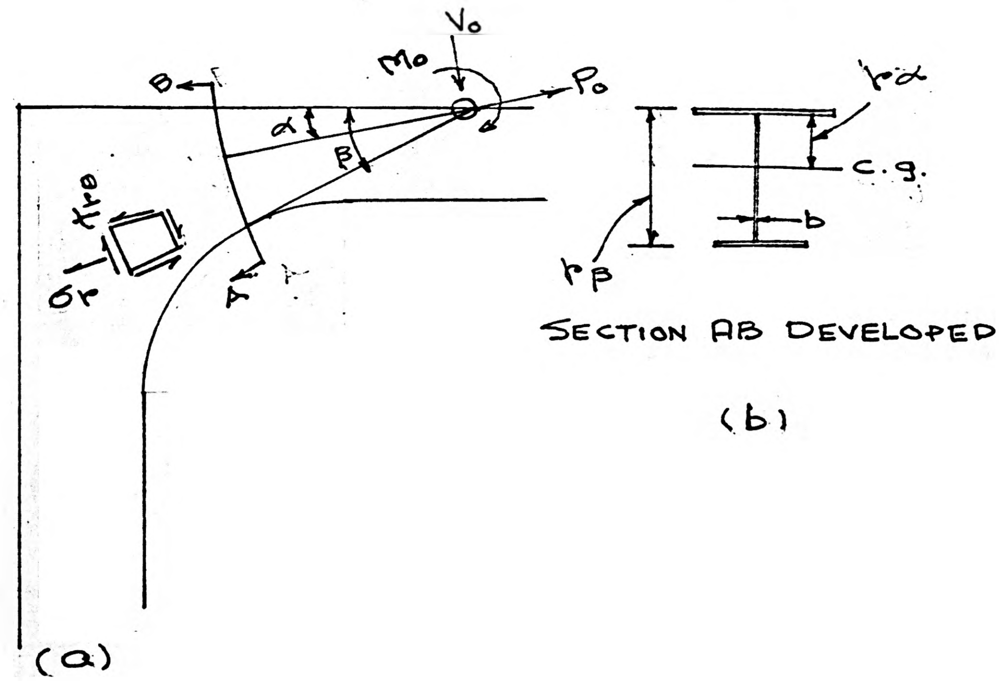
Mr. Hardy Cross and Mr. Newlin Dolbey Morgan give an analysis of haunched beams (1) subjected to bending moments by the method of column analogy. It is shown that the effect of a haunch is to increase the negative bending moment at the supports and to decrease the positive bending moment near the center of the span. The method of column analogy is then applied to find the exact solutions of a haunched single span beam and a continuous beam. Graphs are plotted to give carry-over factors, stiffness factors and fixed end moments for symmetrical, unsymmetrical and straight haunches. The Portland Cement Association has also published A Hand Book Of Frame Constants. (2) These constants are used to determine fixed end moments and stiffness factors for haunched beams.

In their paper "An Investigation of Steel Rigid Frames", (3) Mr. Inge Lyse and Mr. W. E. Black have given a complete distribution of normal and shear stresses at the knee of rigid frames. Two steel frames with built up cross-sections were tested in a 300,000 lb. machine. One frame had a square knee and the other had a curved knee. The clear span was 18 feet. Two sets of strain measurements were taken for both the frames by using strain gauges. One set of measurements was for a constant span length and the other set was taken with a total movement of a quarter inch between supports. The horizontal, vertical and inclined at 45° strain measurements were taken to determine principal stresses at a point. Then equal stress lines corresponding to contour lines were plotted for each knee. It was also found that the horizontal movement of the support does not alter the nature of stress distribution, but the effect is to reduce stresses, as is obvious from the fact that shear and moment are reduced due to partial freedom to rotate.

Mr. Harvey C. Olander in his paper, "A Method for Calculating Stresses in Rigid Frame Corners",[4] gives an analytical procedure to find shear and normal stresses in the corners. The procedure is as follows:

Take a circular section that cuts the extreme fibers at right angles, such as section AB, fig. 1a. Develop the section as shown in fig. 1b and obtain the area "A" and moment of inertia "I" of the developed section. Next resolve all forces to the right of section into forces "P_o", "V_o" and "M_o" about point O, the center of the arc. "P_o" shall pass through center of gravity of section AB, "V_o" shall be normal to "P_o" and "M_o" is the moment of the forces about O. Now with these forces, the stresses on section AB are calculated as for an ordinary beam, except the shear will be determined from "M_o".

In the book Strength of Materials[5] part 2, by S. Timoshenko, a theory for calculating shear in beams



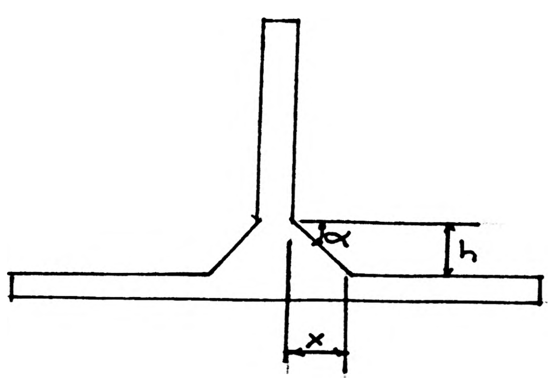
$$\tau_{r\theta} = \frac{VQ}{Ib} = \frac{M_o}{r} \cdot \frac{Q}{Ib}$$

$$\sigma_p = \frac{P_o}{A} \pm \frac{Mc}{I} \quad (M = M_o + V_o p)$$

FIG. NO. 1

of variable cross section has been developed. Here he assumes that the simple beam formula can be used with sufficient accuracy in calculating the normal bending stresses in beams of variable cross section. Then the magnitude of the shearing stresses in this beam can be calculated by applying the same method as used for prismatic beams. He then develops the equation for the shear in such a beam. But here he uses a beam with a cross section symmetrical about the neutral axis.

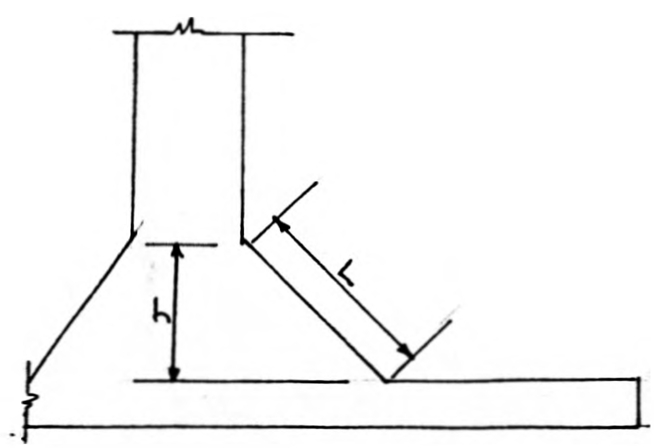
Mr. Richard B. Heagler in his thesis on "Gathering Haunch Design Data by Photoelasticity"[6] gives the nature of shear stresses at a section just outside the point where the haunch intersects the beam. Here he has conducted a photo-elastic investigation on five models, three with x=0.9 inches, and α=30°, 45°, and 60°, one model with h=0.9 inches and α=60°. Then he developed a relationship between the ratio of shear stress as given by $\frac{VQ}{Ib}$ to shear stress as found photoelastically at the same section and the stiffness of the haunch.



III. LIST OF SYMBOLS

Figure 2.

Y axis	--	Face of the column and positive upwards.
X axis	--	Bottom of the beam and positive to the right.
x	--	Distance of any section from the face of the column.
y	--	Distance of the point, at which τ_{xy} for the section considered is maximum from Y axis.
D	--	Depth of any section considered.
d	--	depth of the beam.
τ_{xy} (nom)	-	Maximum shear stress on the section considered.
τ_{xy} (max)	-	Maximum shear stress on a haunch with given X and α.
τ_{xy} beam	--	Shear stress in beam as obtained by using $\frac{VQ}{Ib}$
α	--	Angle of the haunch with horizontal.
Z	--	Stiffness of the haunch given by $Z = \frac{Ch^3}{L}\cos\alpha$
C	--	Constant depending upon the modulus of elasticity of the material.
L	--	Length of the inclined face of the haunch.
h	--	Depth of the haunch.



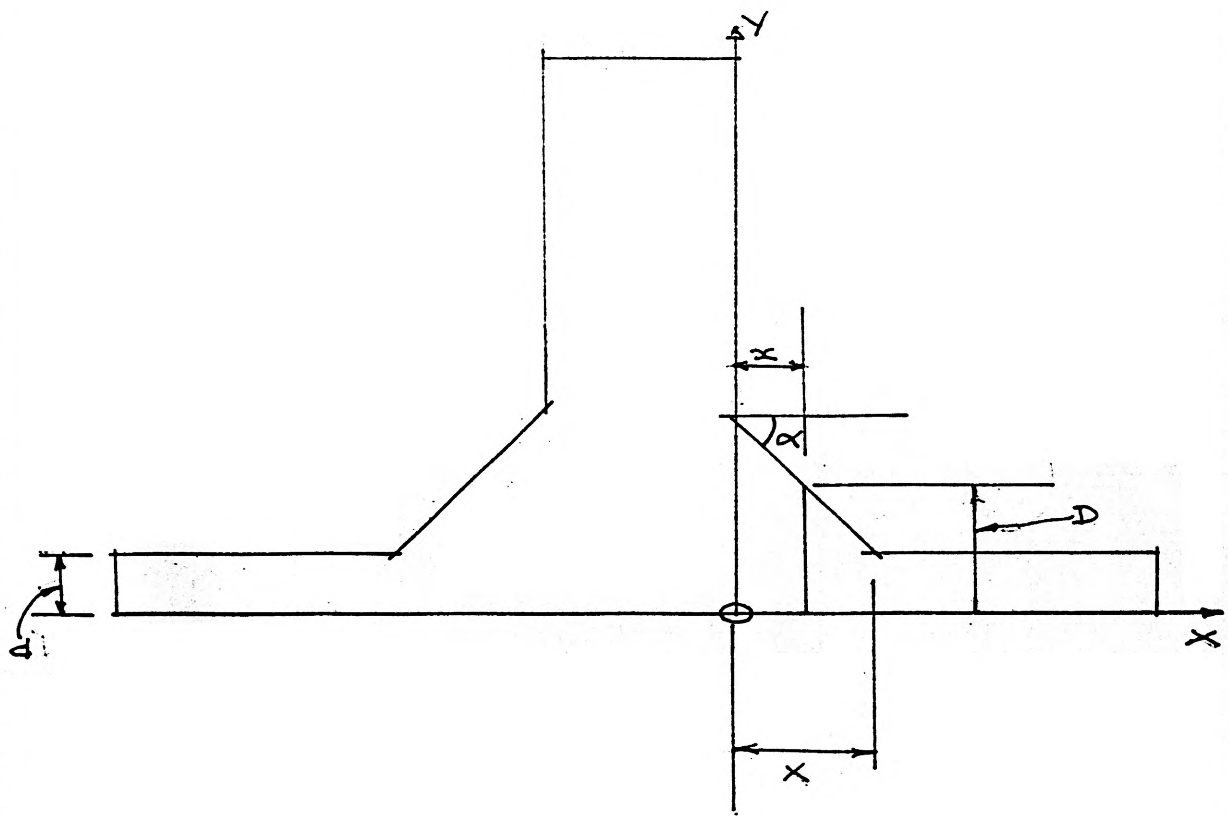


FIG. NO. 2.

n	--	Number of fringes.
F	--	Model fringe value.
f	--	Material fringe value.
t	--	Thickness of the model.
θ	--	Angle that one of the principal stresses make with the horizontal.
p	--	Maximum principal stress.
q	--	Minimum principal stress.

IV. EXPERIMENTAL PROCEDURE

(A) Description of Equipment:

Monochromatic circularly polarized light was used for this experiment. Monochromatic light is used to give clear fringes. No colors are present hence the fringe pattern appears as dark and white bands. Monochromatic light is then circularly polarized so that the isoclinics are removed from the fringe pattern.

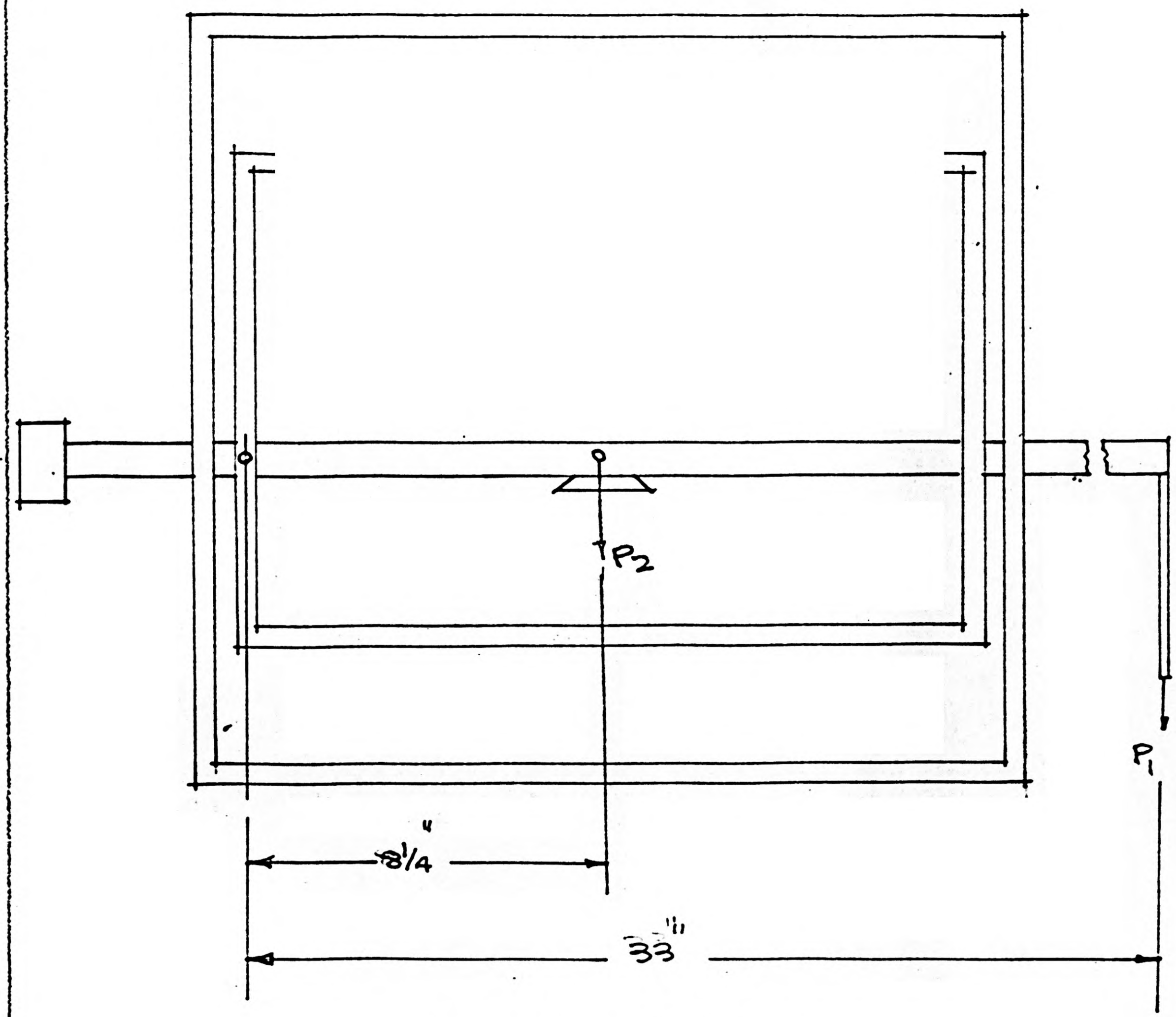
For the purpose of this study a mercury vapor lamp was used with a suitable filter to get monochromatic light. Referring to photograph No. 1, the equipment used in this study was from right to left, a monochromatic light source, polarizer, quarter wave plate, loading device, quarter wave-plate, analyzer and white screen. The sketch of the loading device is shown in the fig. 4. The load is applied through a flat metal bar. The model is so positioned as to obtain a load four times the magnitude of the applied load on the loading bar.

The attached photographs nos. 2,3,4 show the fringe pattern with x=0.7 inches, $=30^{\circ}, 45^{\circ}, 60^{\circ}$. These are typical fringe pattern photographs of sketches nos. 12. Stress calculations are based on the sketches of isoclinics and fringe patterns as obtained from the projected image of the loaded model on the screen.

Polariscope

Photograph No. 1

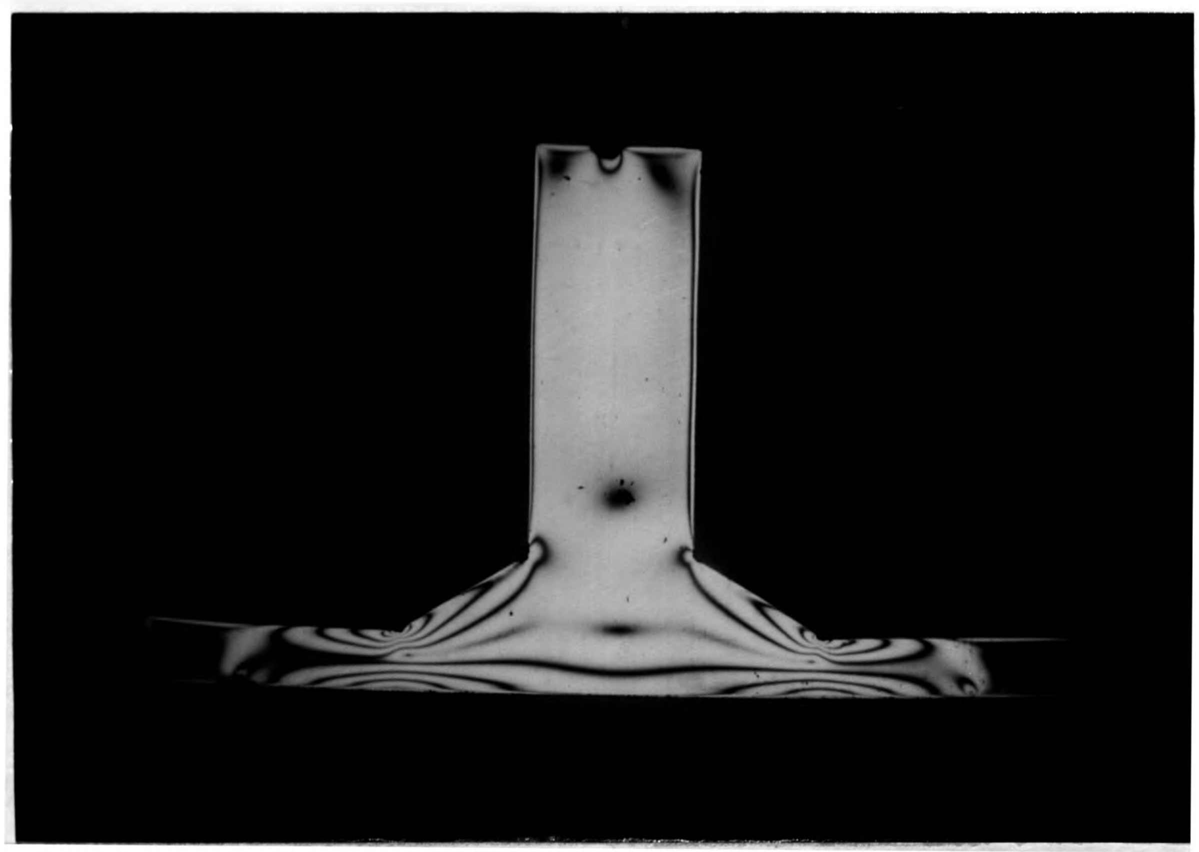
Fig. No. 3



LOADING DEVICE

P_1 - ACTUAL LOAD
P_2 - LOAD APPLIED TO MODEL

FIG. NO. 4

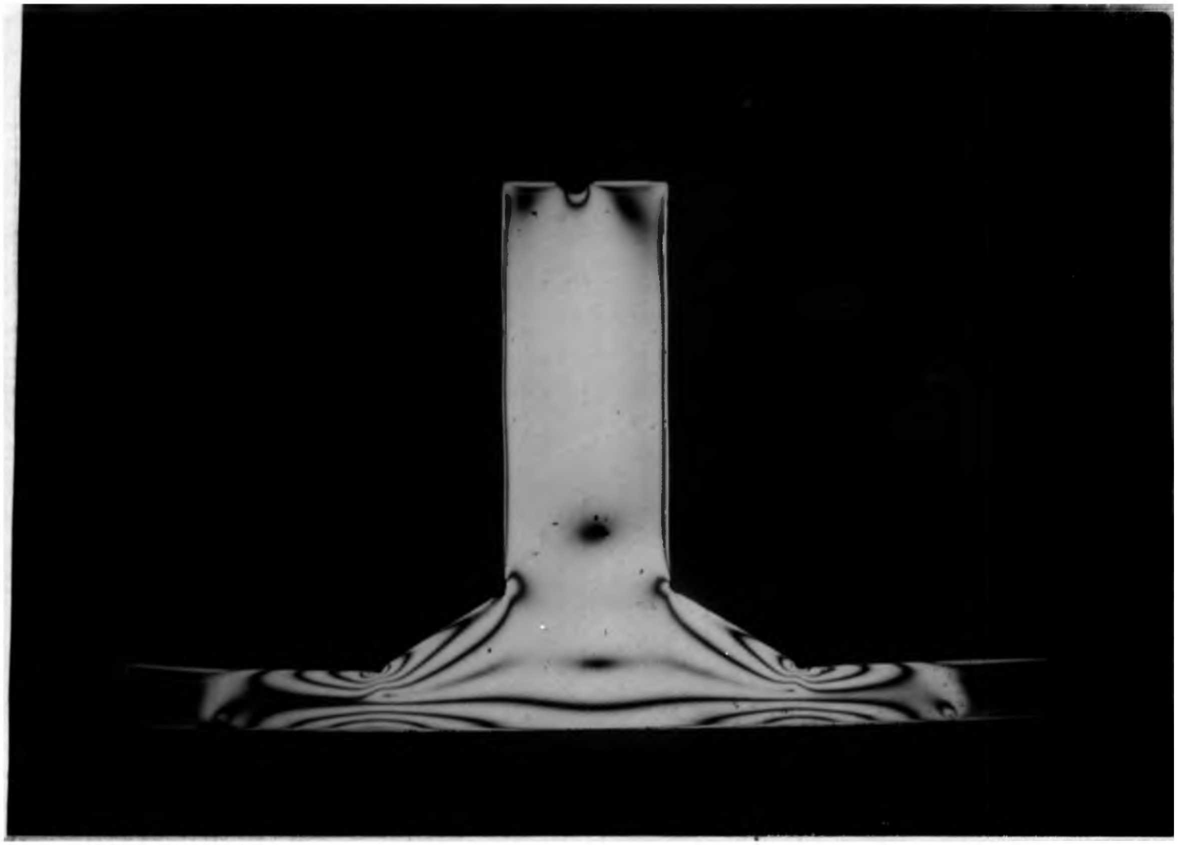


Fringe Photograph No. 2

X 0.7 inches

30

Fig. No. 5

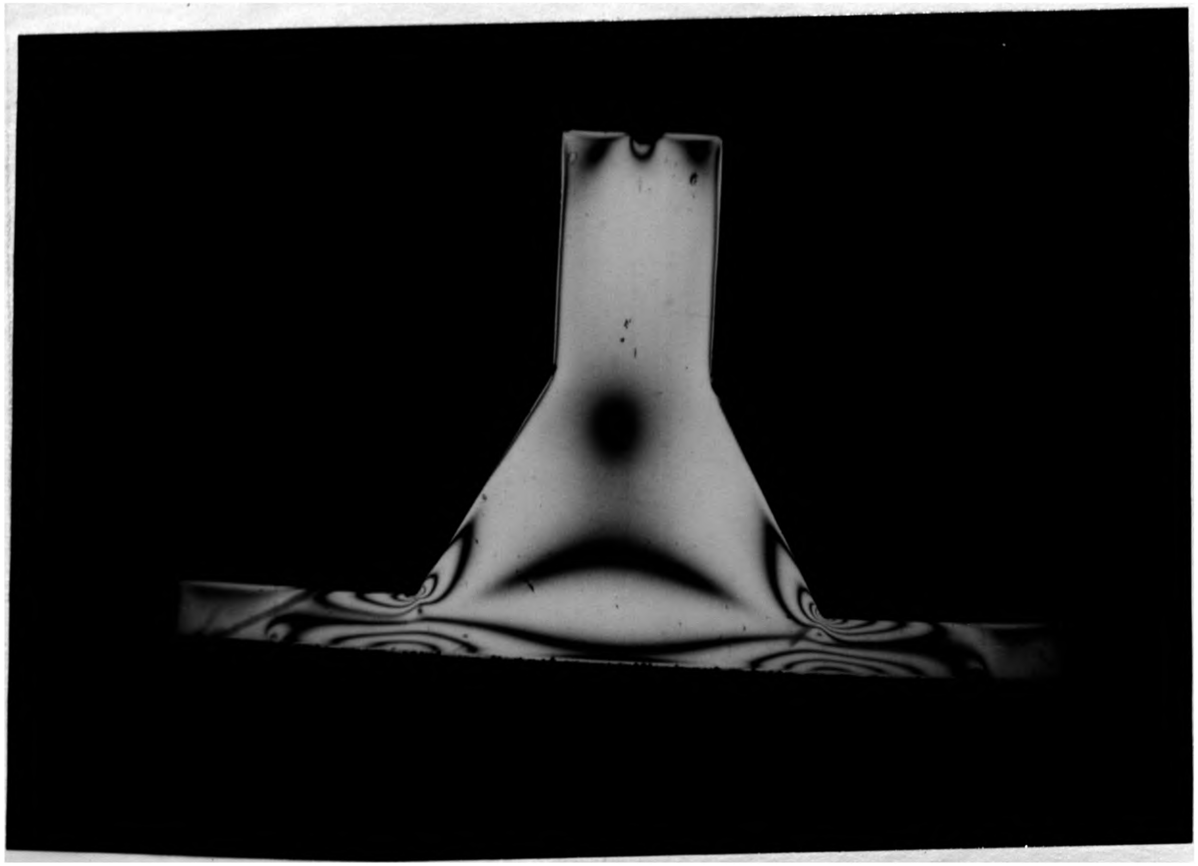


Fringe Photograph No. 3

X 0.7 inches

45

Fig. No. 6



Fringe Photograph No. 4

X 0.7 inches

60

Fig. No. 7

(B) Procedure:

For the purpose of this analysis nine different models were prepared from CR-39.

An aluminum template 1/16 inch thick was prepared as shown in fig. 8 . The various haunches of the same material but of different α and X were prepared as shown in fig. 8 .

All the models were prepared in the following manner:

(1) The basic outline of the beam and column was taped on a CR-39 sheet.

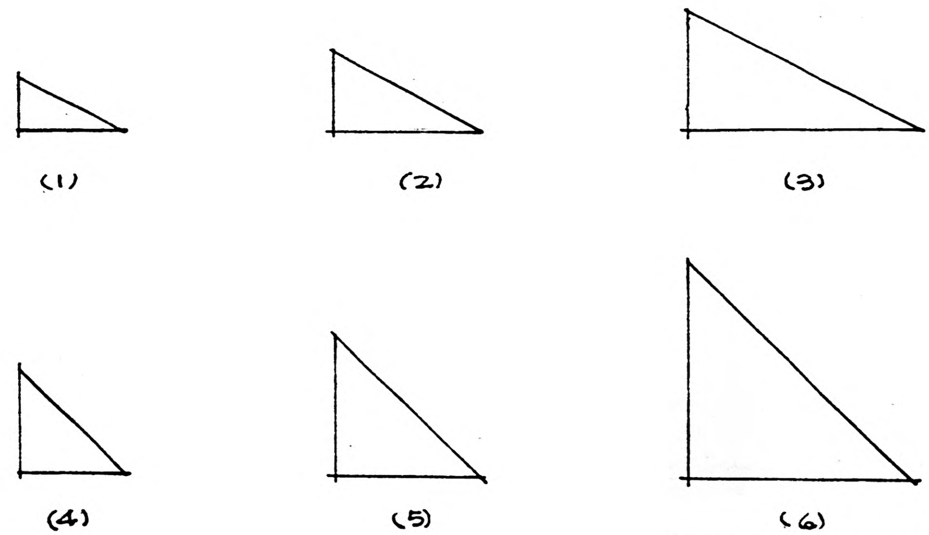
(2) The appropriate haunch template was taped with the beam and column template to the CR-39 sheet. This gave the outline of the model.

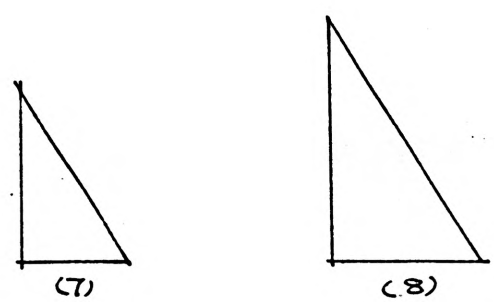
(3) The model was cut with a jig saw to the appropriate shape of a template.

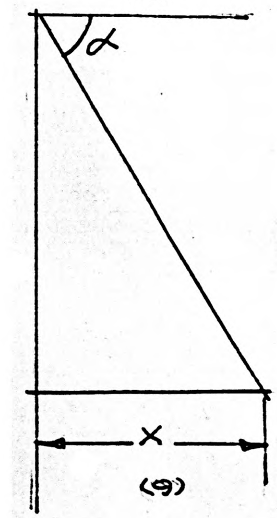
(4) The final shape as guided by the template was machined on the router. The bottom of the bit was smooth so that when the template touched the bit, the grinding of the model was stopped and thus it was possible to get vertical and smooth edges throughout the whole boundary of the model. Fig. 9 .

The edges were machined with the router and the corners were filed simultaneously by hand, but it was not possible to get the edges, which were very near to the corners, as smooth as the edges which were far from corners.

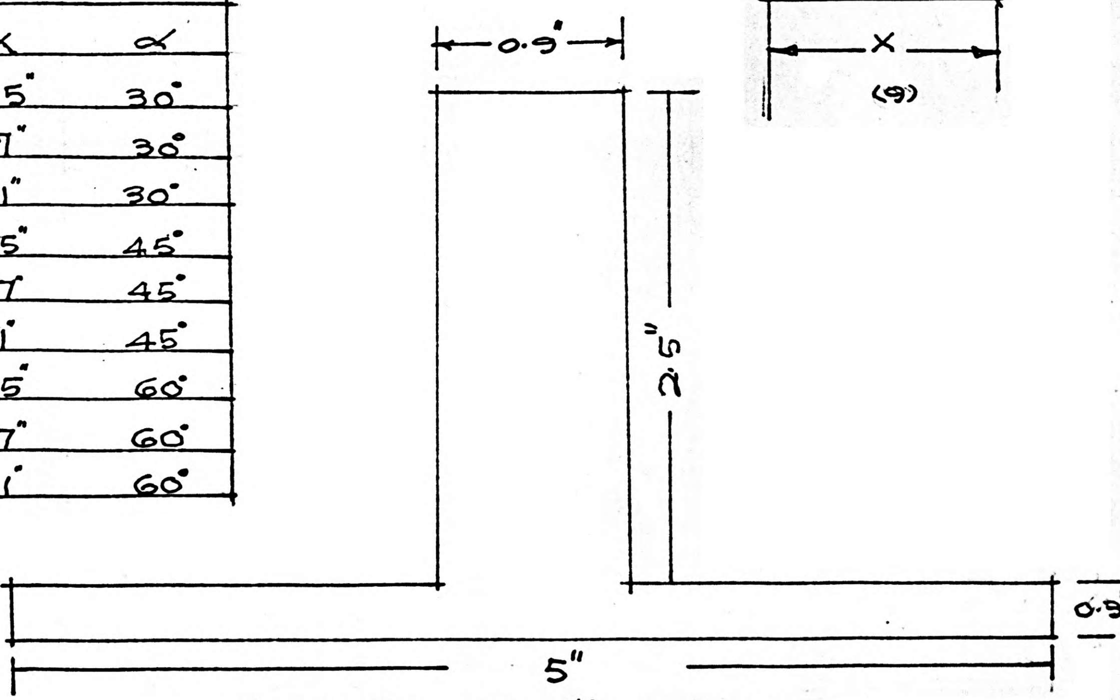
The router was therefore tried for this purpose, and the corners were out by careful sawing, then it was possible





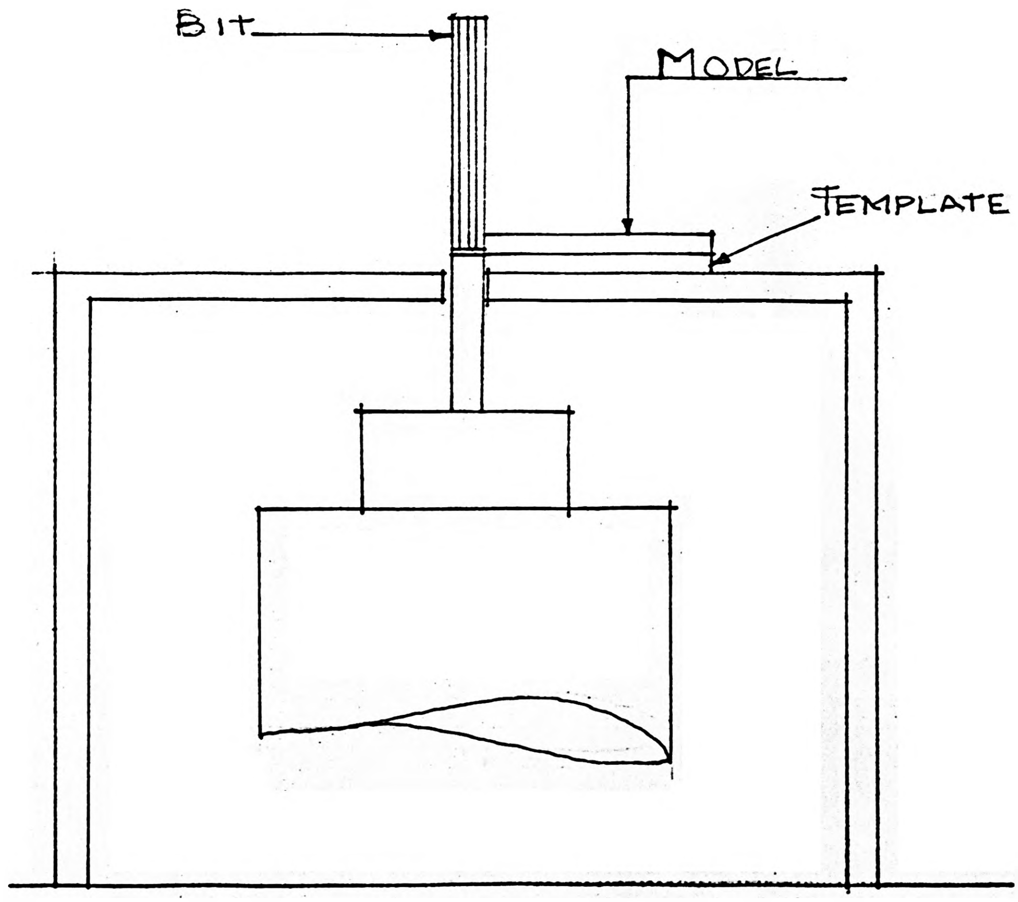


No	X	α
1	0.5"	30°
2	0.7"	30°
3	1.1"	30°
4	0.5"	45°
5	0.7"	45°
6	1.1"	45°
7	0.5"	60°
8	0.7"	60°
9	1.1"	60°



OUTLINE OF TEMPLATES

FIG. NO. 8



ROUTER

FIG. NO. 9.

to get smooth edges. Still in both the methods, machine stresses were induced very near to these points. The errors due to these stresses in the calculations of shear stresses were minimized by taking sections away from these points.

The stresses induced at the corners are local in nature and are probably not too critical when shear stress distribution throughout the haunch is considered. Thus they may be safely ignored in this study.

After the preparation of the model, the testing of the model was conducted immediately to avoid time-edge effect. This testing was done as follows:

(1) The position of the support and the center line of the model was first marked on the model.

(2) Roller supports were fixed on a steel plate at the required distance, namely 4 inches center to center.

(3) The model was placed in a circular polariscope so that marks of the position of the supports on the model coincide with support rollers.

(4) The load was then applied through a roller pin. Extreme care was taken in order to insure that the roller pin was exactly on the center line marked on the model.

(5) The model and the screen were so adjusted as to give a clear image of the model on the screen. These images were approximately three times as large as the model.

(6) All the models were tested at the same load. The total load applied to get fringes was 16 pounds. Monochromatic

light was used to get the fringe patterns. The fringe pattern was then sketched from the projected image of the model, and these sketches were later used in the determination of shear stress distribution.

(7) The source of light was then changed to a white light. In white light the fringes are colored while the isoclinics are black so isoclinics can be easily distinguished. Also as the isoclinics are independent of the load, by increasing or decreasing the load a clear black line, or an isoclinic, can be easily obtained. These isoclinics were also sketched on the sheet of paper. An effort was made to obtain the isoclinics at intervals of 15 degrees, but in some sketches 0 and 5 degrees isoclinics are also sketched to get better results.

Sometimes it was very difficult to get clear isoclinics. In this case to separate the isoclinics from the fringes, the applied load was increased or decreased and the resulting changes could be clearly seen on the screen. Thus the method of projecting the image on the screen and then sketching the isoclinics and fringes was of great help when all the operations were performed by one man.

Prior to the calculations of shear stresses across the sections, the material fringe value for CR-39 was found by the following procedure:

A beam six inches by one and one-quarter inches was was cut from the 1/4 inch thick sheet of CR-39, which was used for the models. This beam was then subjected to pure bending. The loading of this beam was carried out as shown in fig. 10 .

The load was applied gradually and the load at which each full fringe appeared at the edge was noted. When the beam is in pure bending, there are no shear stresses across the section and hence all the stresses at any section of the beam are principal stresses. If the edge is considered, then the stress perpendicular to the boundary is zero and thus there is only one principal stress, which may be either compressive or tensile, so the fringe at the boundary gives the value of principal stress directly.

A graph of fringe number vs. applied load was plotted as shown in graph no. 1 . The material fringe value was found as follows:

$$\tau_{max} = nF \qquad \text{(1)}$$

$$\tau_{max} = \frac{p-q}{2} \qquad \text{(2)}$$

At the boundary one of the principal stresses is equal to zero, therefore

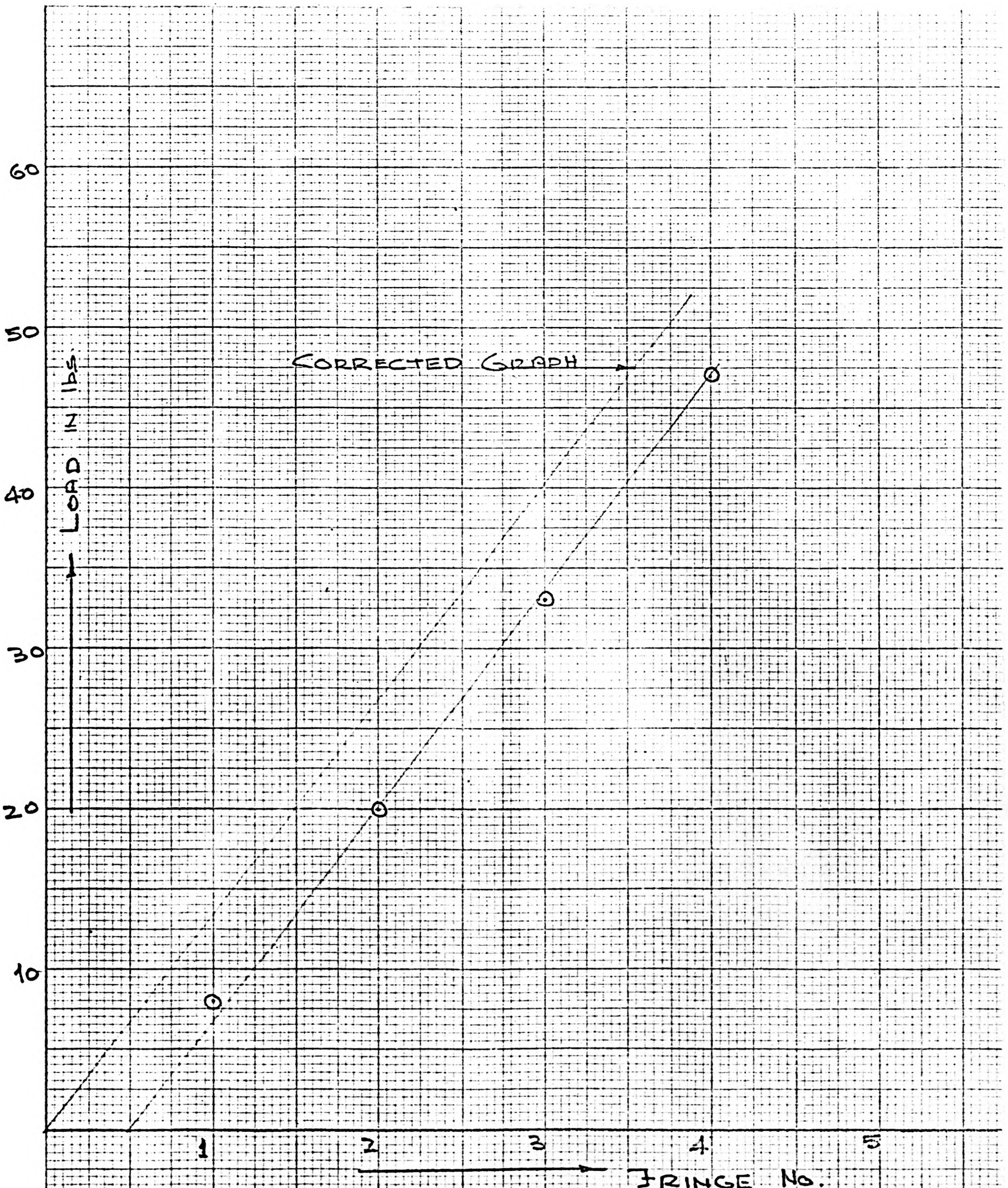
$q = 0$ at the bottom of the beam.

From (2)

$$\tau_{max} = \frac{p}{2} \qquad \text{(3)}$$

Equating (1) and (3)

$$nF = \frac{p}{2}$$



Actual load = (LOAD x 4)

LOAD - FRINGE GRAPH

GRAPH. No. 1

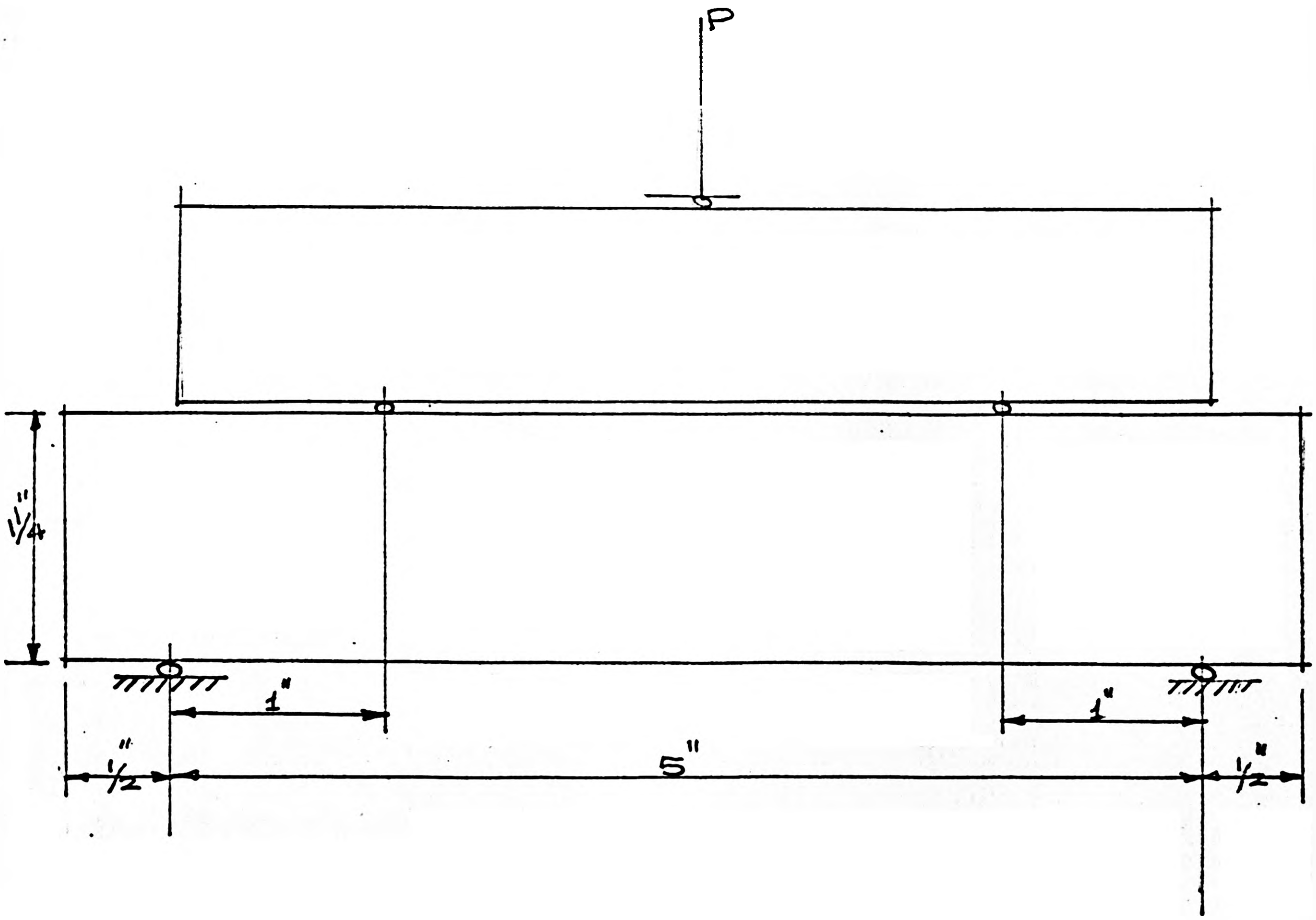


FIG. NO. 10

$\therefore \quad p = 2nF$

$\therefore \quad F = \dfrac{p}{2n}$ ____________ (4)

$f = Ft$ ____________ (5)

Tensile stresses are considered positive while compressive stresses are considered negative.

Choose any value of load and find the corresponding 'n' from the corrected graph. Then the stress at the bottom of the beam as calculated from the flexure formula $\dfrac{My}{I}$ gives p, where

M - applied moment

y - distance of the point considered, from neutral axis

I - moment of inertia of the section

$p = \dfrac{My}{I}$

Applied load = 108#

Then from graph n=2

Applied moment $M = \dfrac{108}{2} \times 1 = 54$ in. lbs., fig. 10 .

$y = \dfrac{1.25}{2} = 0.625$ inches

$I = \dfrac{bd^3}{12}$

b = 0.25 inches

d = 1.25 inches

$I = \dfrac{bd^3}{12} = 0.0406 \quad (\text{inch})^4$.

$p = \dfrac{My}{I}$

$= \dfrac{54 \times 0.625}{0.406}$

But $F = \frac{p}{2n}$ from equation (4)

$$= \frac{54 \times 0.625}{0.0406} \times \frac{1}{2 \times 2}$$

$$F = 207.85$$

$f = Ft$ from equation (5)

$$= 207.82 \times 0.25$$

$= 51.95$ psi. in shear.

V. STRESS CALCULATIONS

The face of the column is taken as the ordinate and is positive upwards. The bottom of the beam is taken as the abscissa and is taken positive to the right as shown in fig. 11 . The first section was taken 0.2 inches from the face of the column and successive sections were taken at 0.2 inches increments. The last section was taken at 0.1 inches outside the haunch corner.

For example if one considers a haunch of width 0.7 inches then sections at 0.2, 0.4, 0.6 and 0.8 inches from the face of the column were taken and shear stress distribution on all the sections was found, thus giving a complete shear stress distribution throughout the haunch.

The reason for not selecting a section at the corner was to avoid machining stresses at the corner. As stated before, it was not possible to avoid completely the machining stresses at these sharp corners. Hence to minimize their effect as much as possible, sections just outside and just inside the haunch were considered.

The next step was to find the shear stress distribution on each section. This was done as follows:

(1) Each section was marked on the sketch of isoclinics and fringes according to its magnification ratio. For example if a sketch of fringes shows that the sketched image is three times the size of the model, then 0.2 inches section will be at a distance of 0.2 x 3.00 = 0.6 in.

from Y-axis. The same procedure was followed for the isoclinic sketch. Generally this ratio is the same for both isoclinics and fringes because both were sketched at the same time.

(2) Each section was divided into ten equal parts along the Y-axis.

(3) A graph of $\frac{p-q}{2}$ vs. vertical distance from the bottom of the beam was drawn.

(4) A graph of θ vs. the vertical distance from the bottom of the beam was drawn.

(5) Using the relation

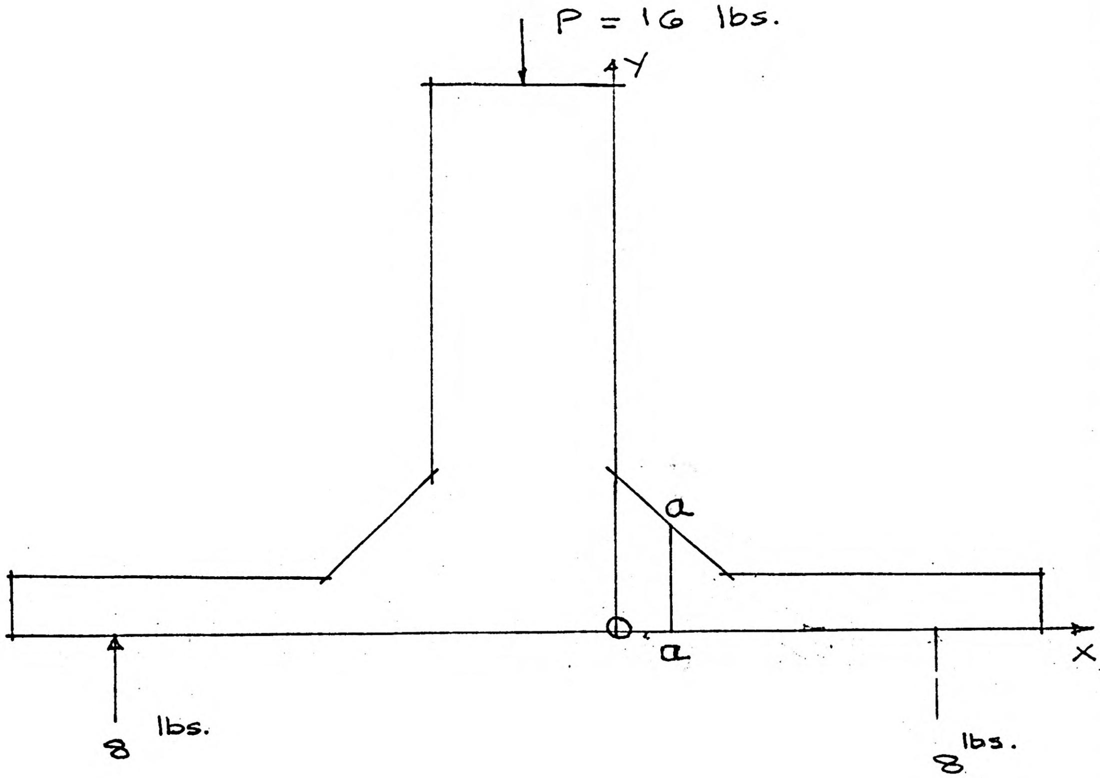
$$\tau_{xy} = \frac{p-q}{2} \sin 2\theta = nF \sin 2\theta$$

the shear stress at each of the ten points was calculated.

(6) The graph of τ_{xy} vs. distance from the bottom of the beam was drawn.

(7) The total load applied to each model was 16 pounds, hence each reaction equals 8 pounds. Now if a free body diagram of the model at a particular section is considered, the external shear stress must be balanced by internal shear stress. Thus the net area under the τ_{xy} curve times the thickness of the model must represent 8 pounds for equilibrium. Fig. 11 .

The τ_{xy} curve is first drawn through all the points obtained mathematically. Then the area under the τ_{xy} curve was measured by a planimeter. If this area times the thickness represented a force equal to 8 pounds, or



FOR EQUILIBRIUM

$C = T \quad \& \int \tau_{xy}\, dA = 8^{\#}$

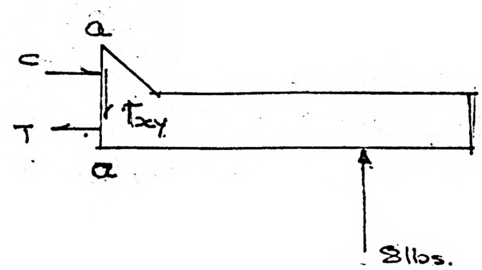


FIG. No. 11

if the difference between this measured force and the applied shear force of 8 pounds was less than 10 percent, no adjustment in the curve was made, but if the difference between the above two quantities is more than 10 percent, then the τ_{xy} curve was adjusted so as to give a deviation of less thab 10 percent.

The adjustment of the curve was done by drawing a mean curve through the points. This adjustment was done in the middle portion of the section. In this position the isoclinics are close and so there is every possibility of making a mistake in reading the angle θ.

To make the above procedure clear, the actual mathematical calculations as carried out for all the sections are represented by the following calculations:

<u>Section bb</u> - for a haunch with $x = 0.7$ inch $\alpha = 30^{\circ}$

Magnification ratio

$$= \frac{\text{Depth of the beam on sketch}}{\text{Actual depth of the beam}}$$

Actual depth of the beam $= d = 0.3''$

$$= \frac{0.9}{0.3}$$

$$= 3$$

0.2" on model equals to $0.2 \times 3 = 0.6$ inches on the sketch.

Similarly 0.4" = 0.4 x 3 = 1.2 inches

0.6" = 0.6 x 3 = 1.8 inches

0.8" = 0.8 x 3 = 2.4 inches

These are then marked on the sketch. Each section is then divided into ten equal parts and the values of shear stress on each of these points is found by using

$$\tau_{xy} = \frac{p-q}{2} \sin 2\theta$$

$$= nF \sin 2\theta$$

d	n	2θ	Sin 2θ			τ_{xy}
0.1d	2.9	0	Sin 0		0.00	0.00
0.2d	2.2	4	Sin 4		0.10	0.22
0.3d	1.65	14	Sin 14		0.242	0.400
0.4d	1.05	38	Sin 38		0.585	0.615
0.5d	0.10	106	Sin (90 +16)	Cos 16	0.96	0.096
0.6d	0.50	160	Sin (90 + 70)	Cos 70	0.342	0.171
0.7d	1.80	194	Sin (180 + 14)	-Sin 14	-0.242	-0.435
0.8d	2.50	216	Sin (180 + 36)	-Sin 36	-0.615	-1.54
0.9d	2.90	230	Sin (180 + 50)	-Sin 50	-0.765	-2.22
1.00d	3.20	240	Sin (180 + 60)	-Sin 60	-0.866	-2.76

The value of τ_{xy} as obtained from the above table was then plotted. The area on the left hand side of the ordinate axis is considered as negative and that on the right hand side as positive.

Then positive and negative areas under the τ_{xy} curve is found by planimeter as follows:

Positive area

Initial reading of planimeter = 3600

Final reading of planimeter = 3677

Difference 77

Negative area ----

Initial reading = 3600

Final reading = 3913

Difference = 313

Net area under τ_{xy} curve

$$= 313 - 77$$

$$= 236$$

Now 101 divisions on planimeter represents 1 square inch.

236 division $= \frac{236}{101}$ square inches

Depth of the section considered

$$= \frac{\text{total depth as measured on sketch}}{\text{magnification ratio}}$$

$$= \frac{1.05}{3} = 0.35 \text{ inches}$$

If the scales of the graphs are examined carefully, then it will be seen that

1/2 inch on X axis = 0.5 fringes

1/2 inch on Y axis = 0.1d

and thus, 1/4 square inch

= (0.5 x 0.1d) fringe inches

This when multiplied by (F x t)

= (0.5 x 0.1d) x 207.85 x 1/4 lbs.

1 square inch

$$= 0.5 \times 0.1d \times 207.85 \times \frac{1}{4} \times 4 \text{ lbs.}$$

= (10.20 d) lbs.

Total lbs represented by 236 divisions

$$= \frac{236}{101} \times 10.20 \times 0.35$$

= 8.35 lbs.

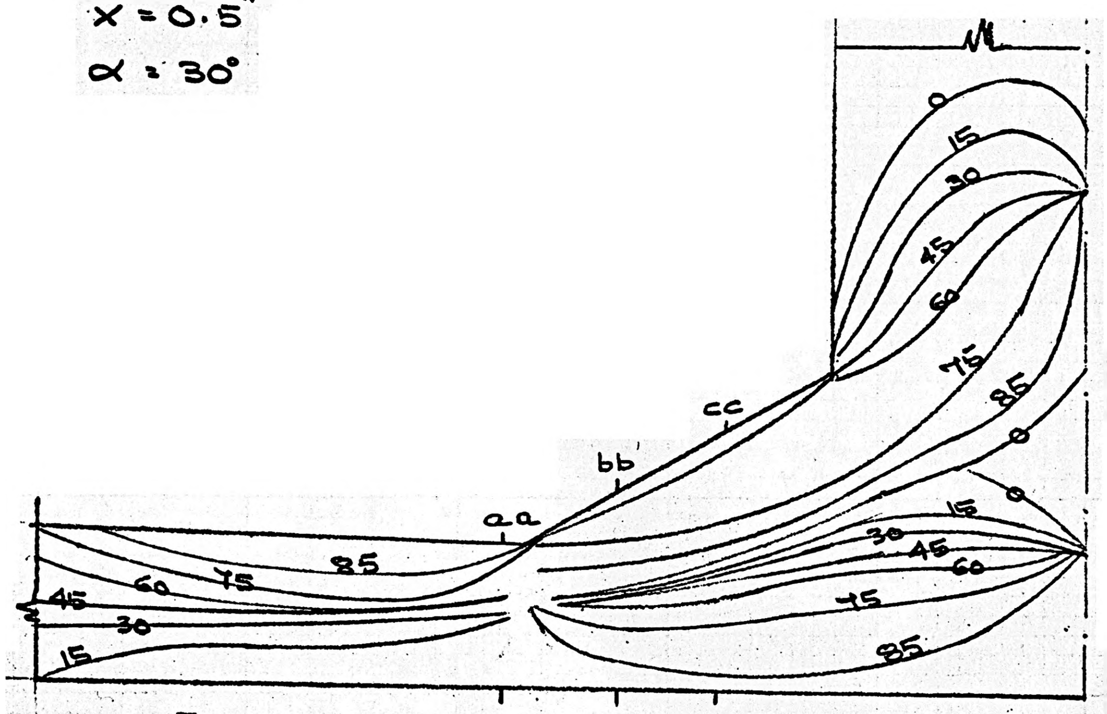
But applied shear = 8 lbs.

$$\% \text{ deviation} = \frac{8 - 8.35}{8} \text{ lbs.} \times 100$$

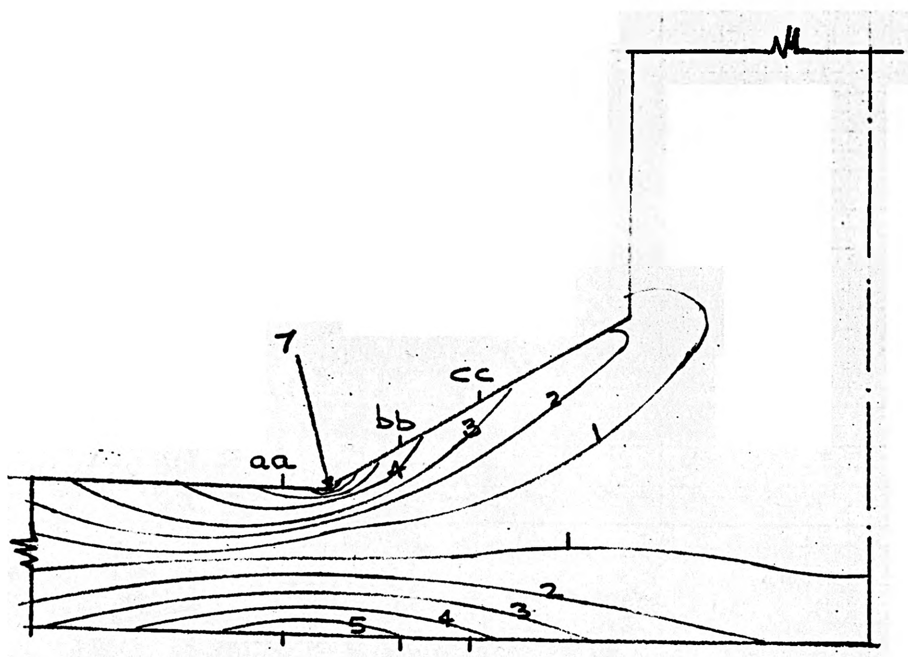
$$= -4.37\ \%$$

$x = 0.5''$

$\alpha = 30^\circ$

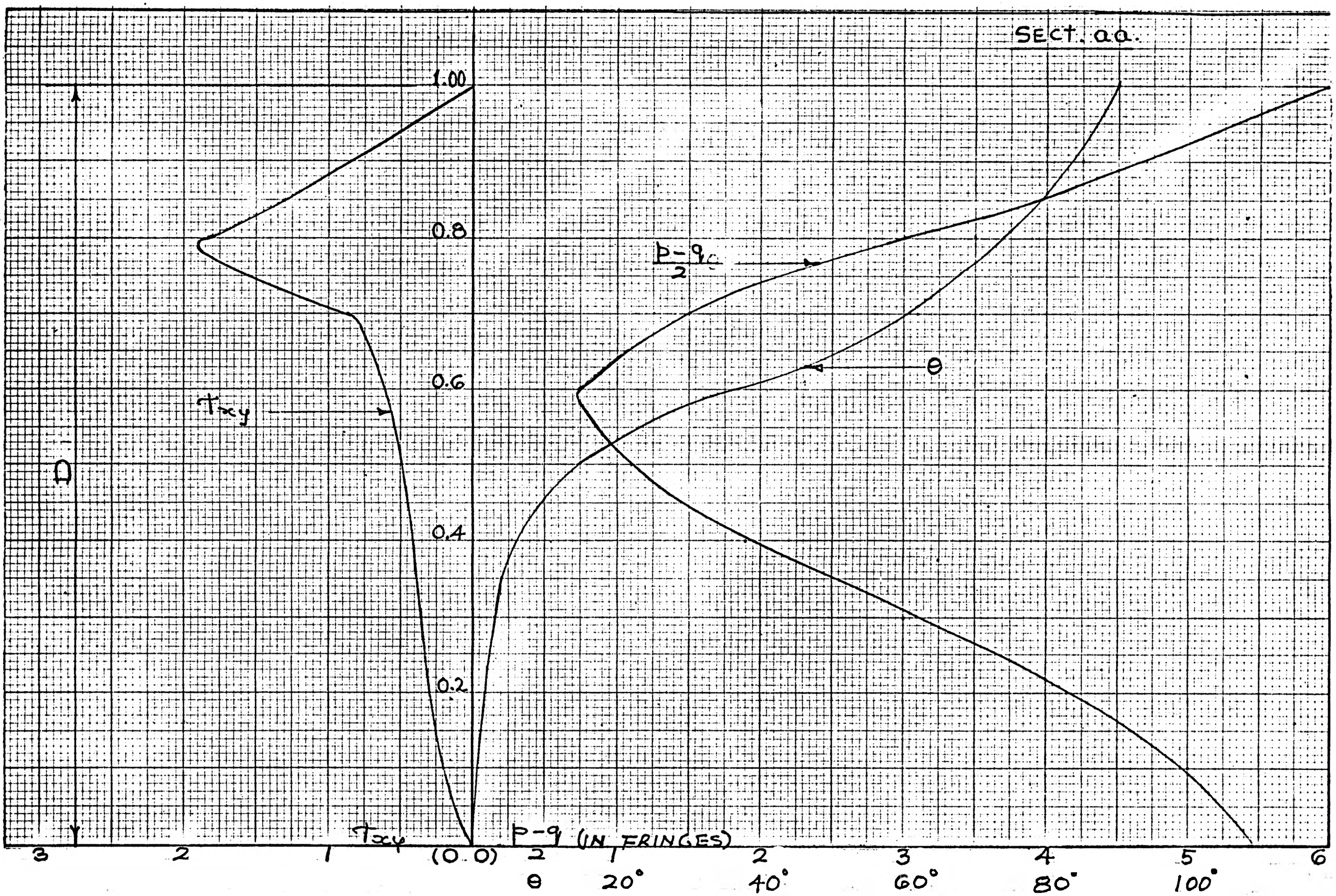


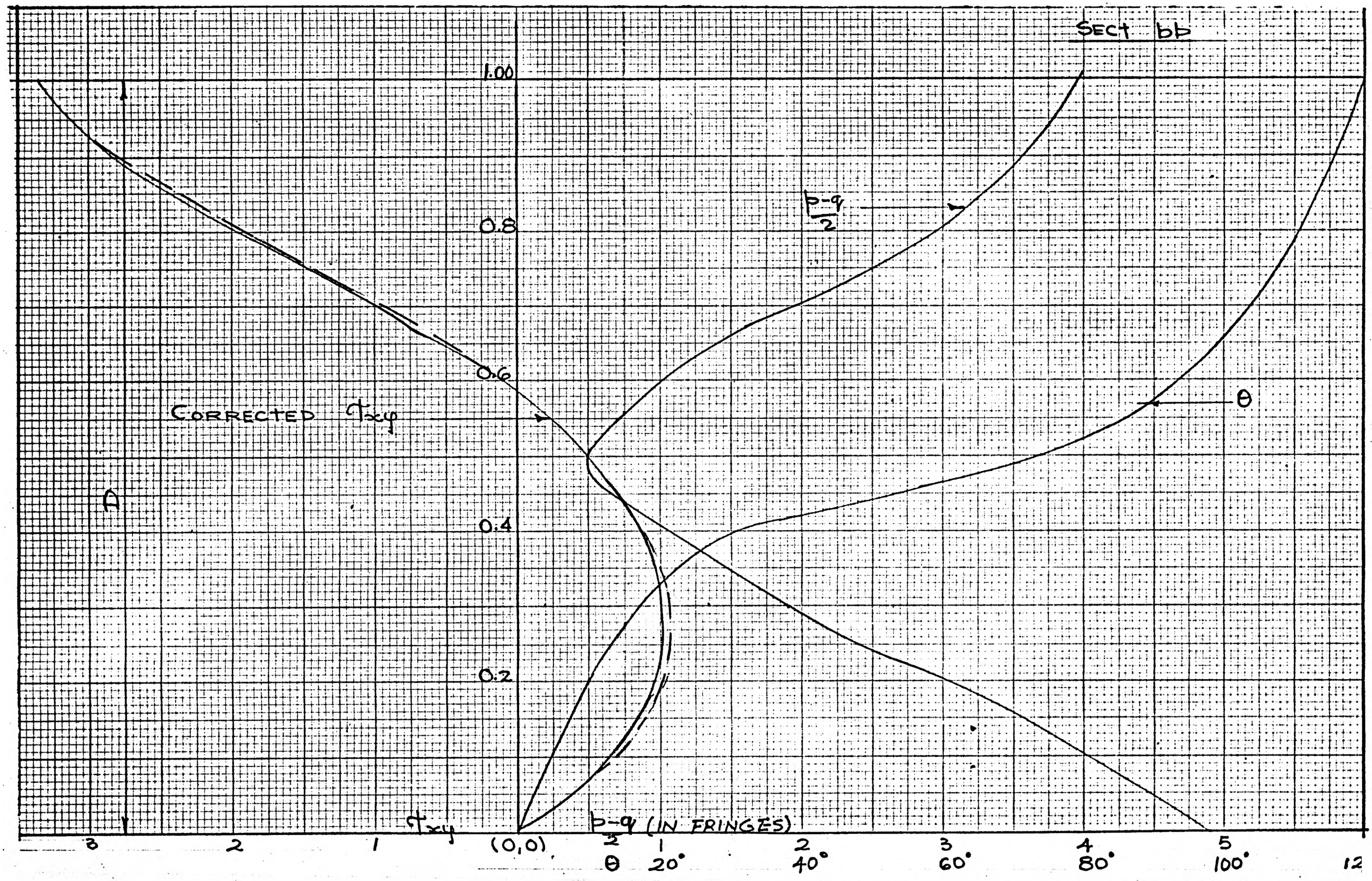
Isoclinics



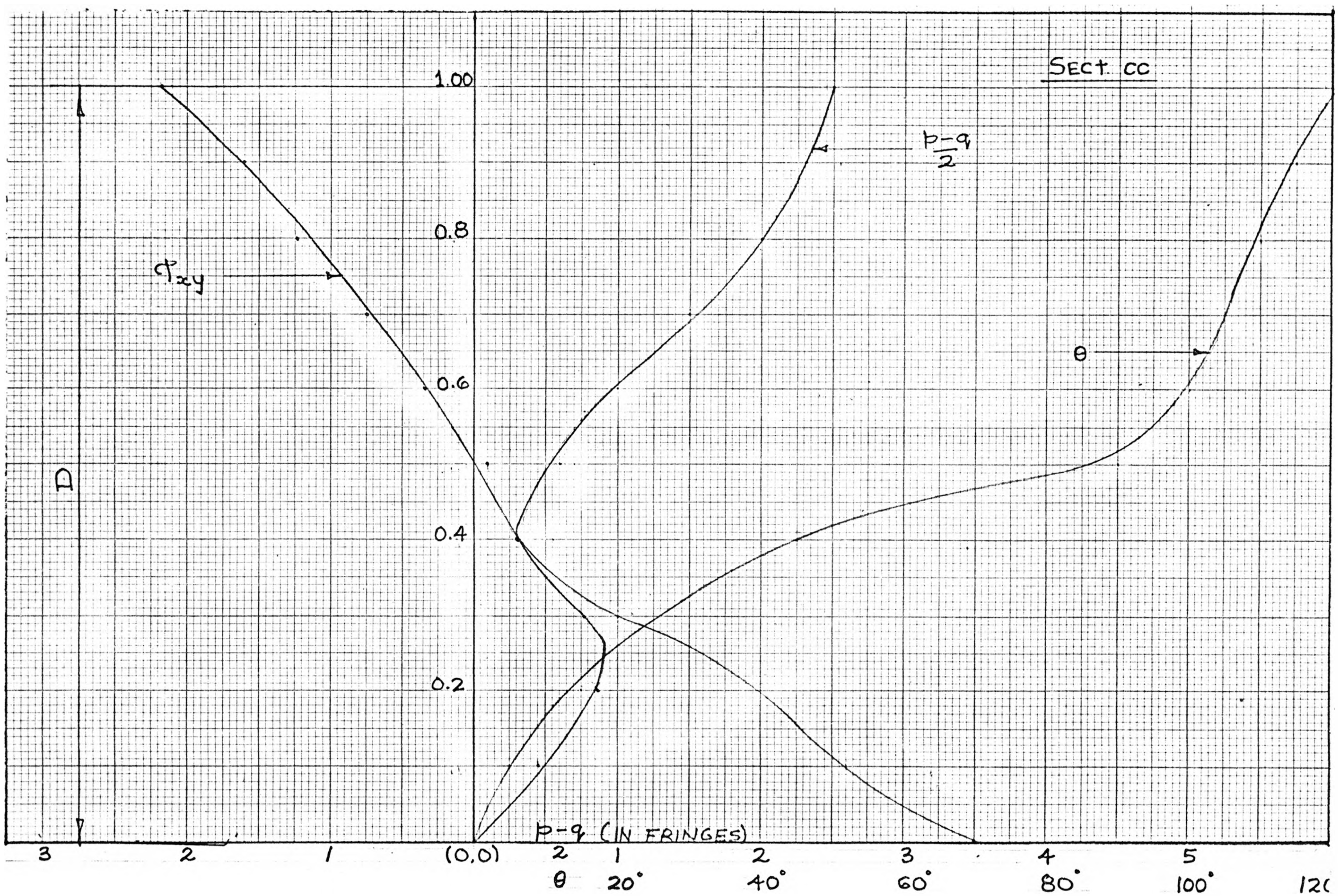
Fringe Pattern

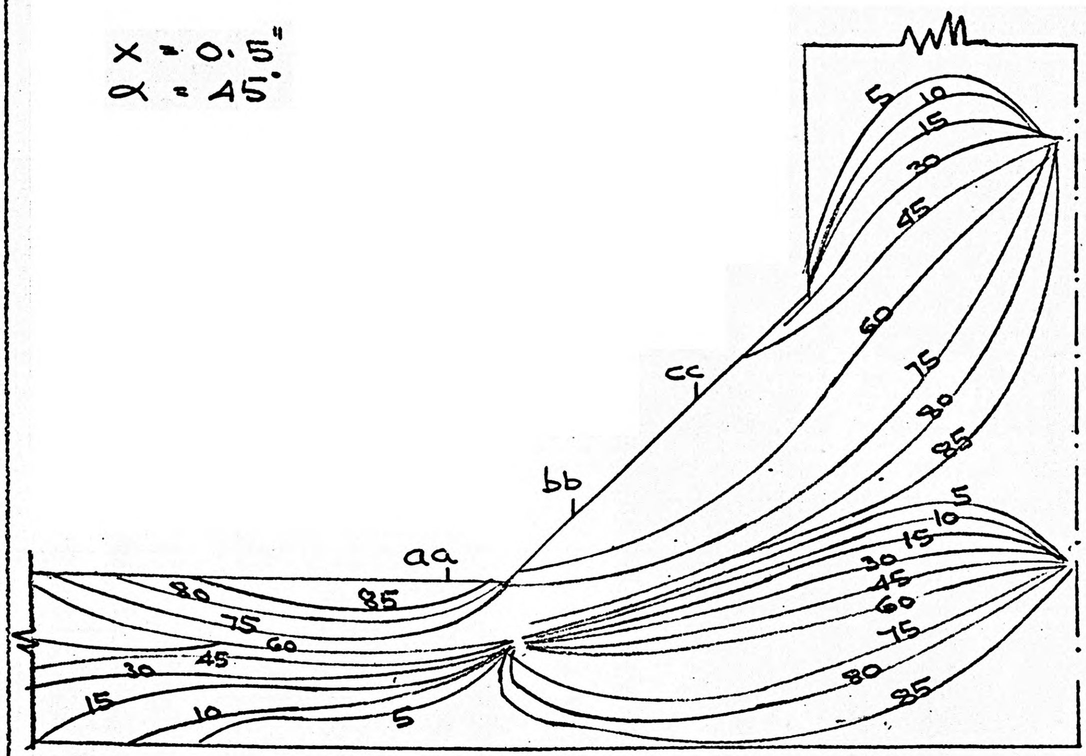
Fig. 12


SECT. aa.
1.00
0.8
0.6
0.4
0.2
D
τ_{xy}
$\frac{p-q}{2}$
θ
3
2
1
τ_{xy}
(0.0)
$\frac{p-q}{2}$ (IN FRINGES)
θ
1
2
3
4
5
6
20°
40°
60°
80°
100°

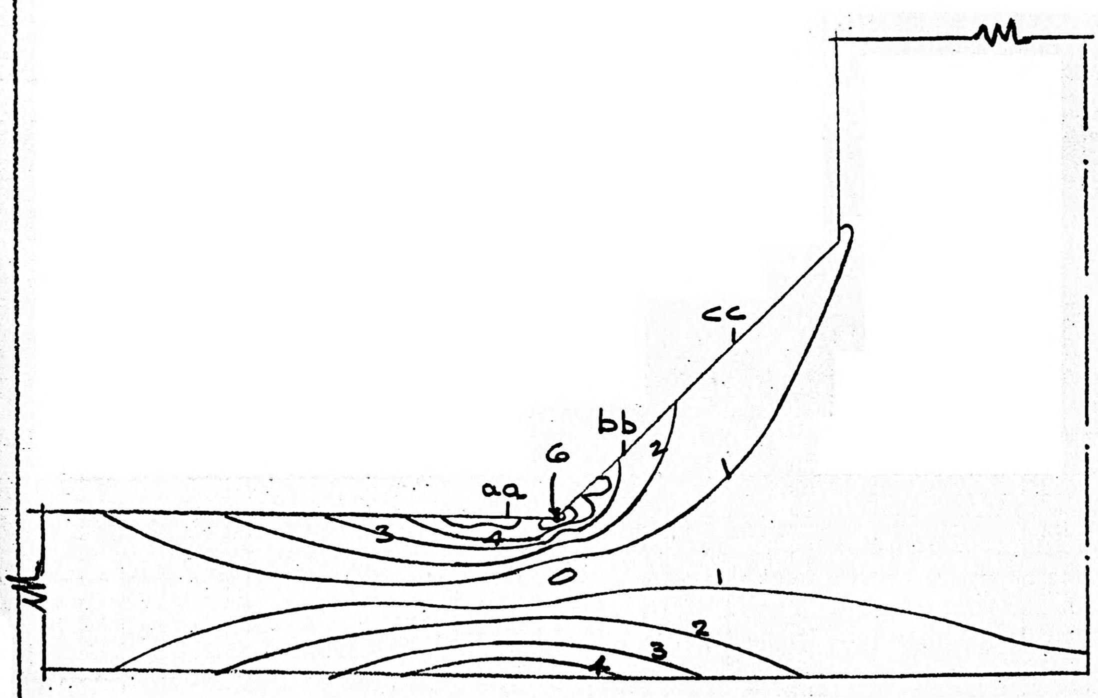


SECT bb
1.00
0.8
0.6
0.4
0.2
p-q / 2
θ
CORRECTED τxy
A
τxy
(0,0)
p-q / 2 (IN FRINGES)
θ
3
2
1
1
2
3
4
5
20°
40°
60°
80°
100°
12


Sect cc
1.00
0.8
0.6
0.4
0.2
τ_{xy}
$\frac{p-q}{2}$
θ
D
p-q (IN FRINGES)
3
2
1
(0,0)
2
1
2
3
4
5
θ
20°
40°
60°
80°
100°
12(

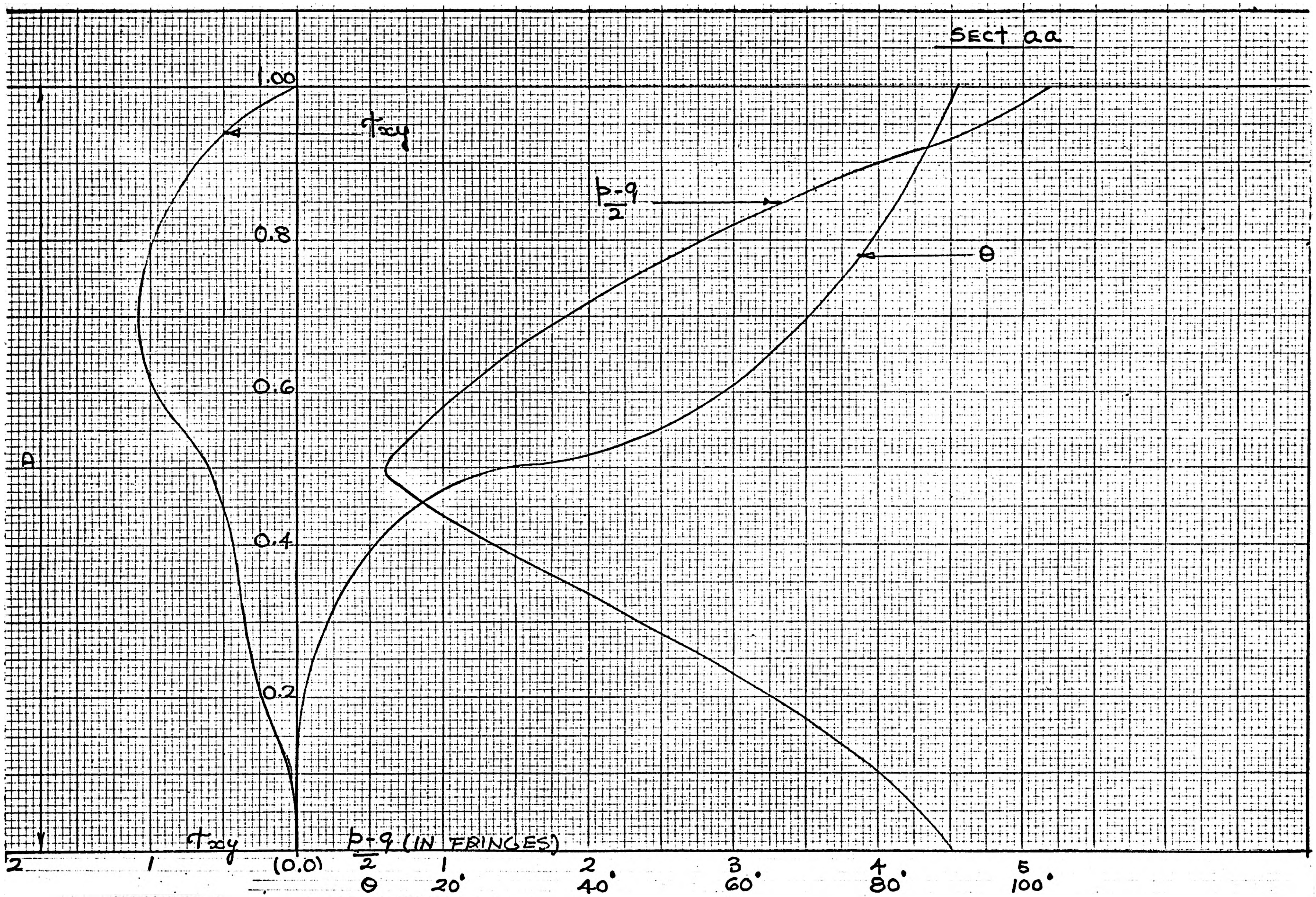


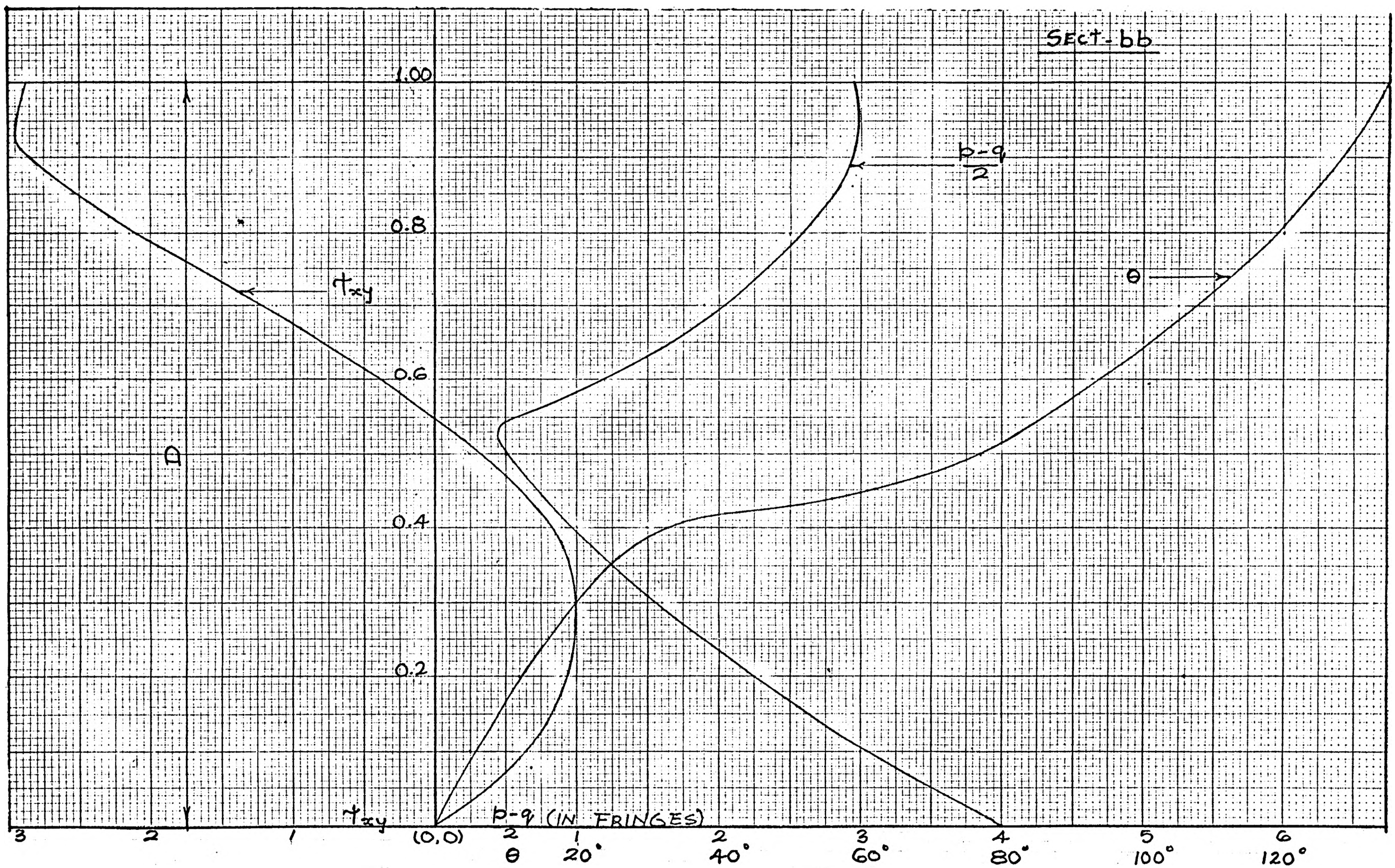
ISOCLINICS

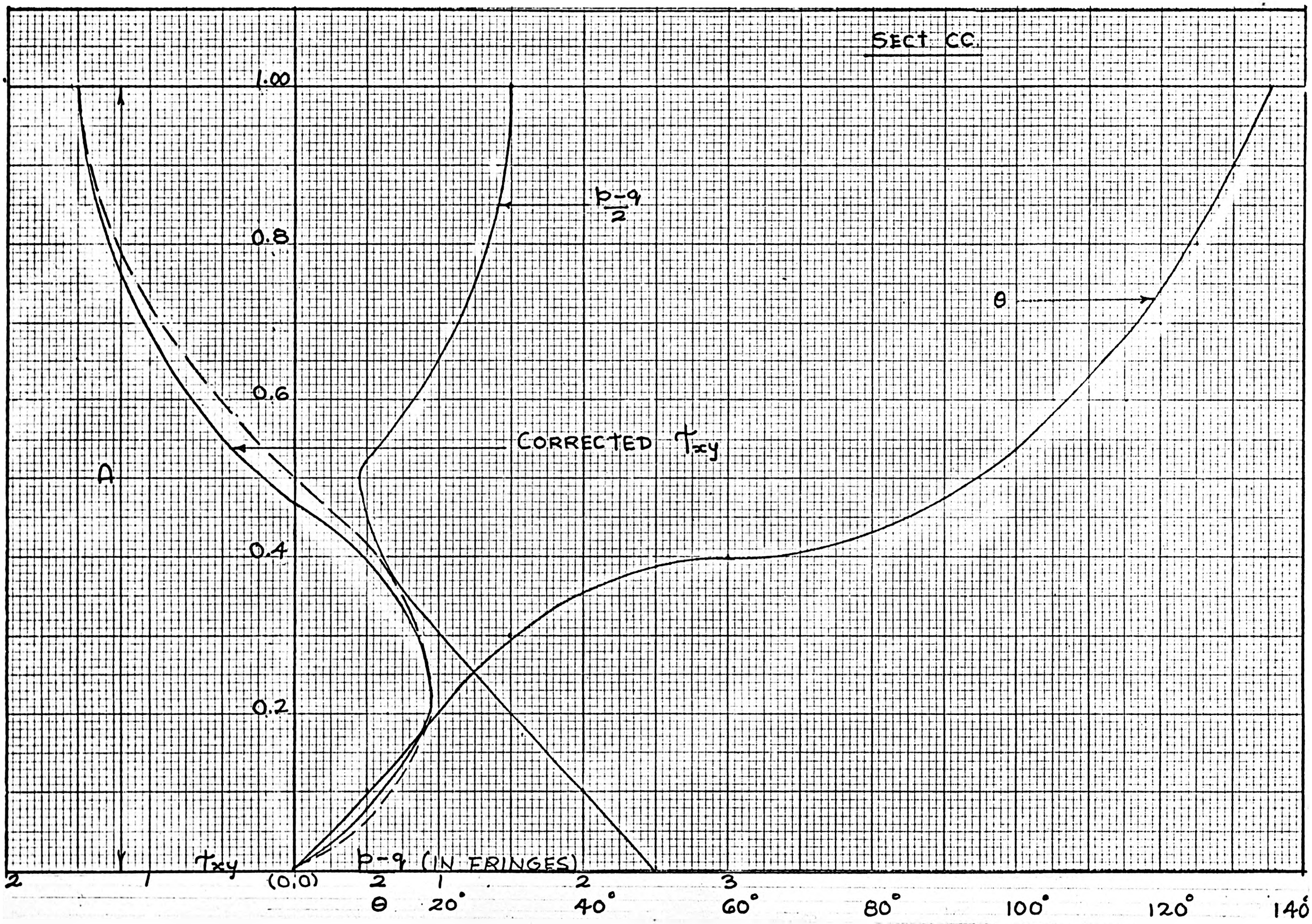


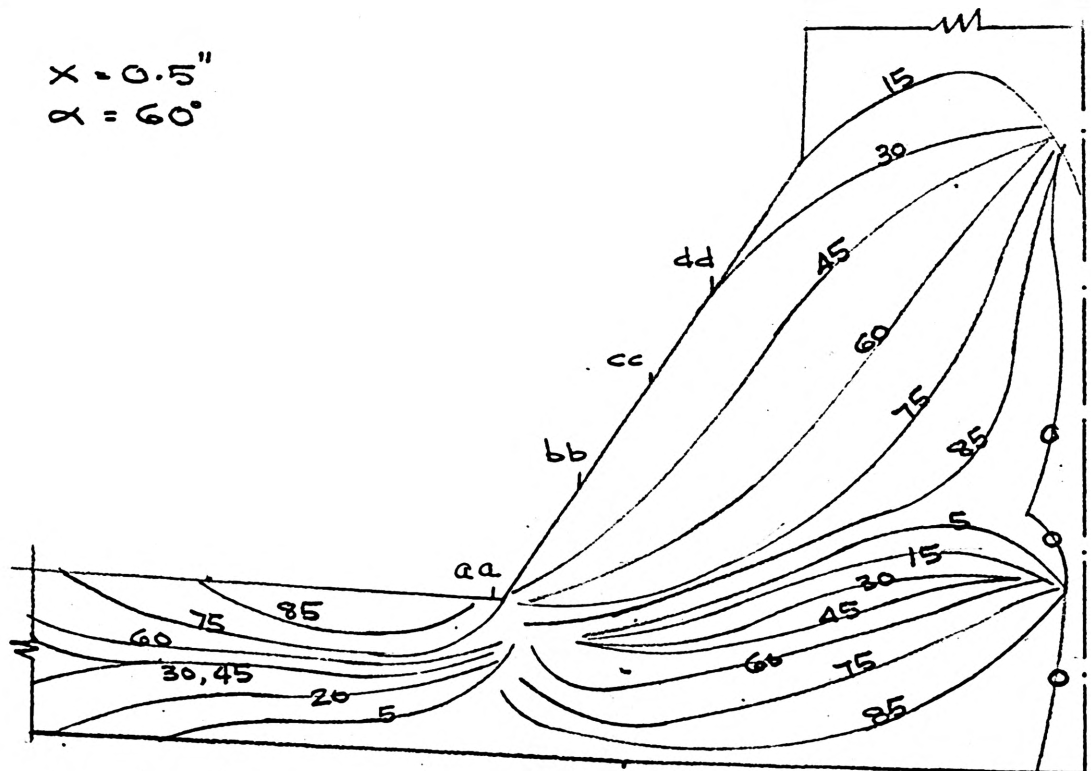
FRINGE PATTERN

FIG. 12

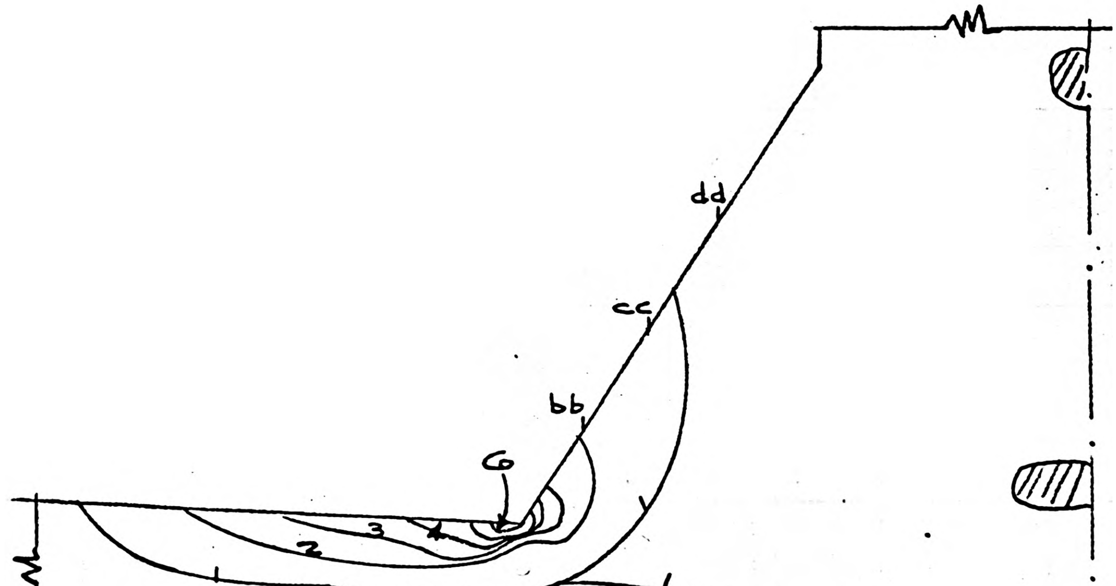

SECT aa
1.00
0.8
0.6
0.4
0.2
A
τ_{xy}
$\frac{p-q}{2}$
θ
τ_{xy}
2
1
(0.0)
$\frac{p-q}{2}$ (IN FRINGES)
θ
1
20°
2
40°
3
60°
4
80°
5
100°


SECT - bb
1.00
0.8
0.6
0.4
0.2
$\frac{p-q}{2}$
τ_{xy}
θ
τ_{xy}
(0,0)
p-q (IN FRINGES)
θ
3
2
1
2
1
2
3
4
5
6
20°
40°
60°
80°
100°
120°


SECT CC
1.00
0.8
0.6
0.4
0.2
$\frac{p-q}{2}$
θ
CORRECTED τ_{xy}
A
τ_{xy}
p-q (IN FRINGES)
2
1
(0,0)
2
1
2
3
θ
20°
40°
60°
80°
100°
120°
140°

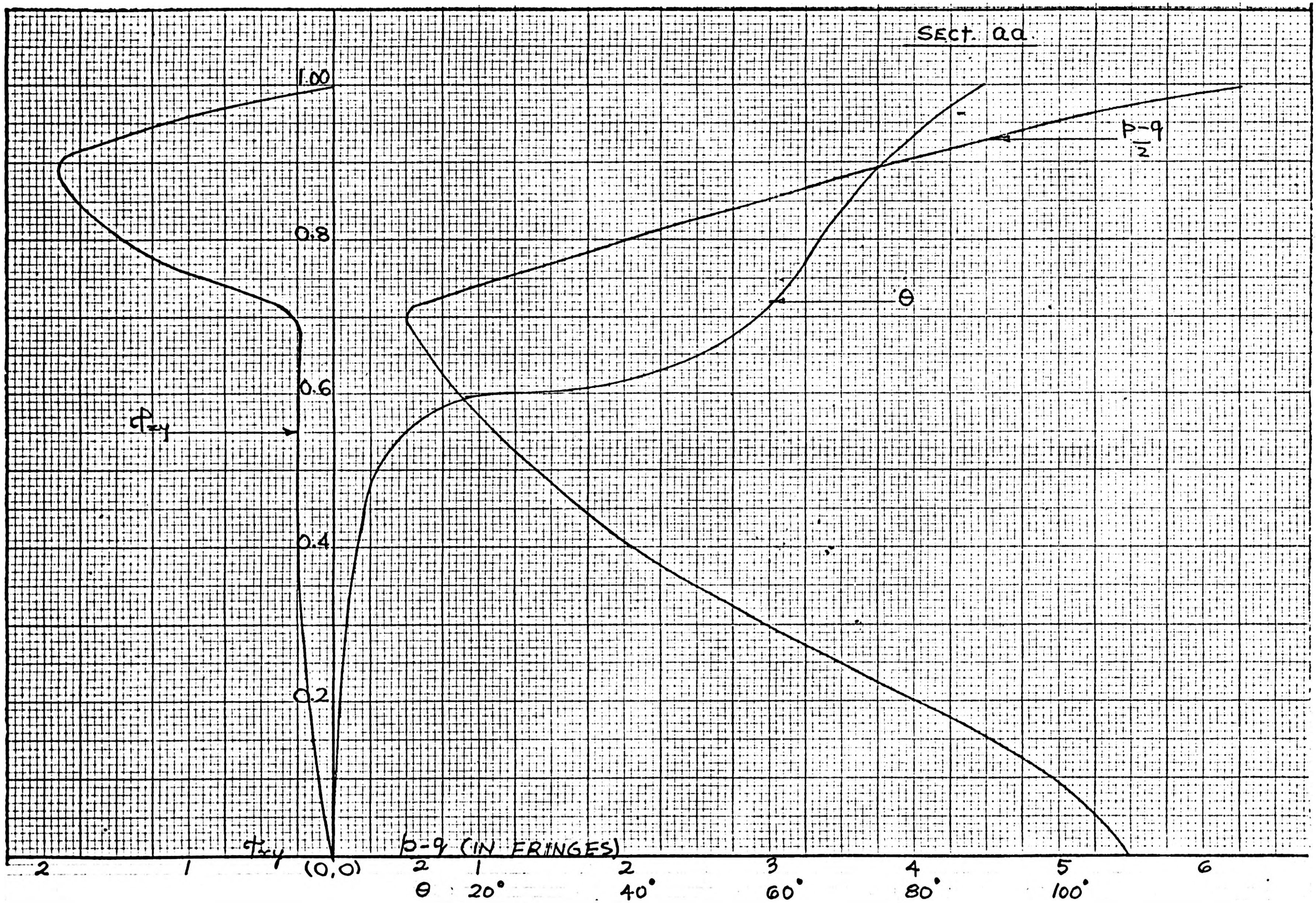


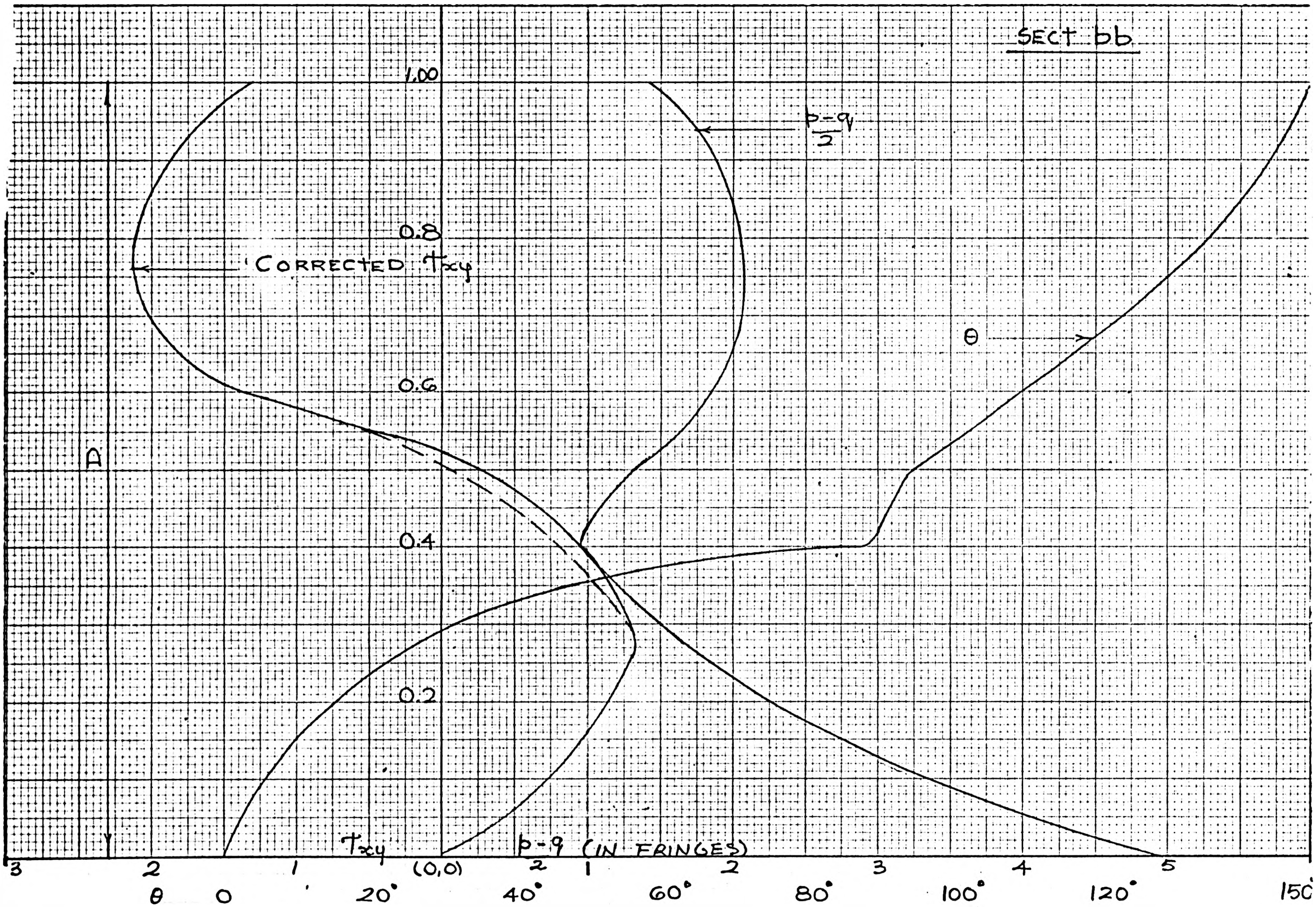
ISOCLINICS

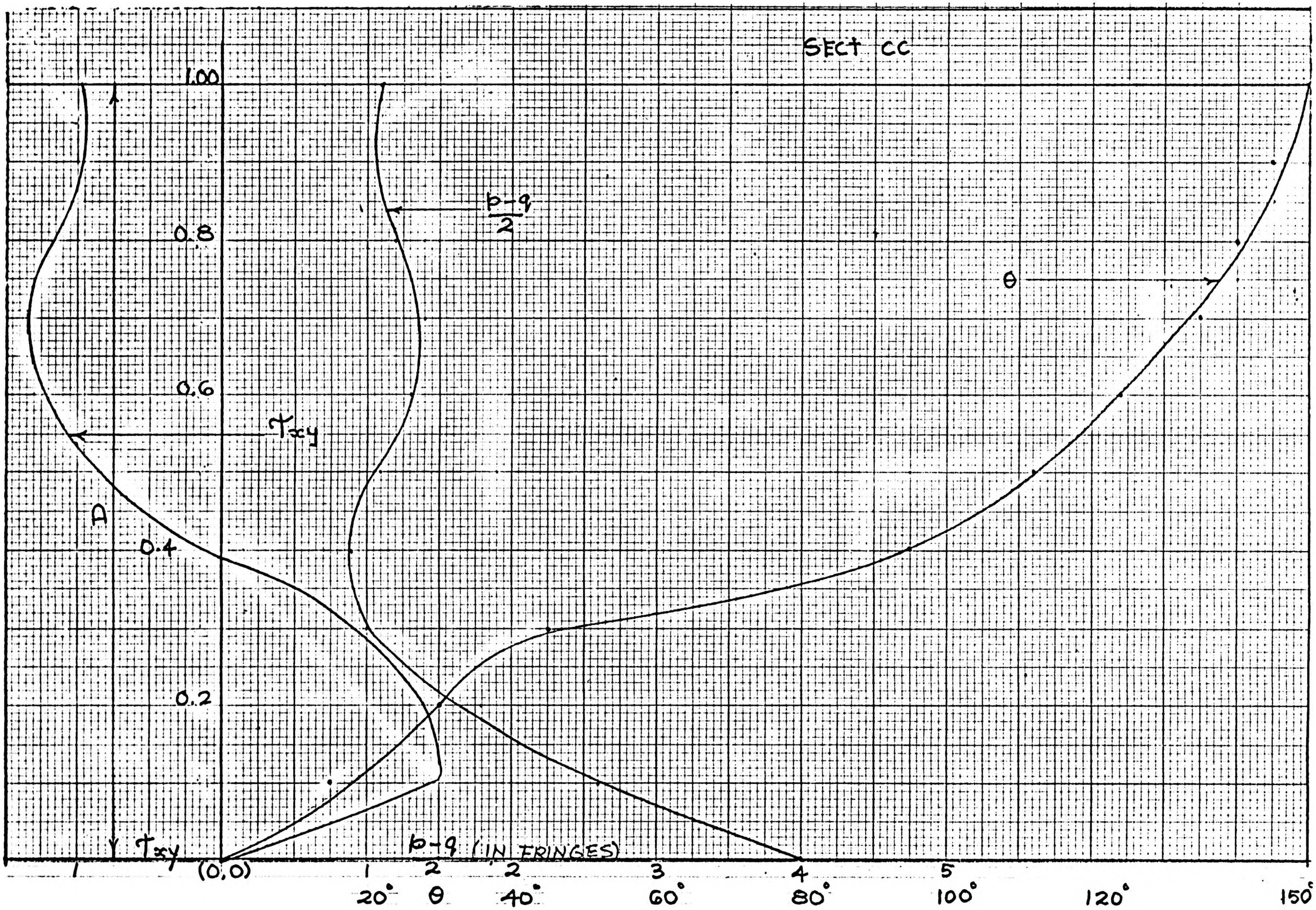


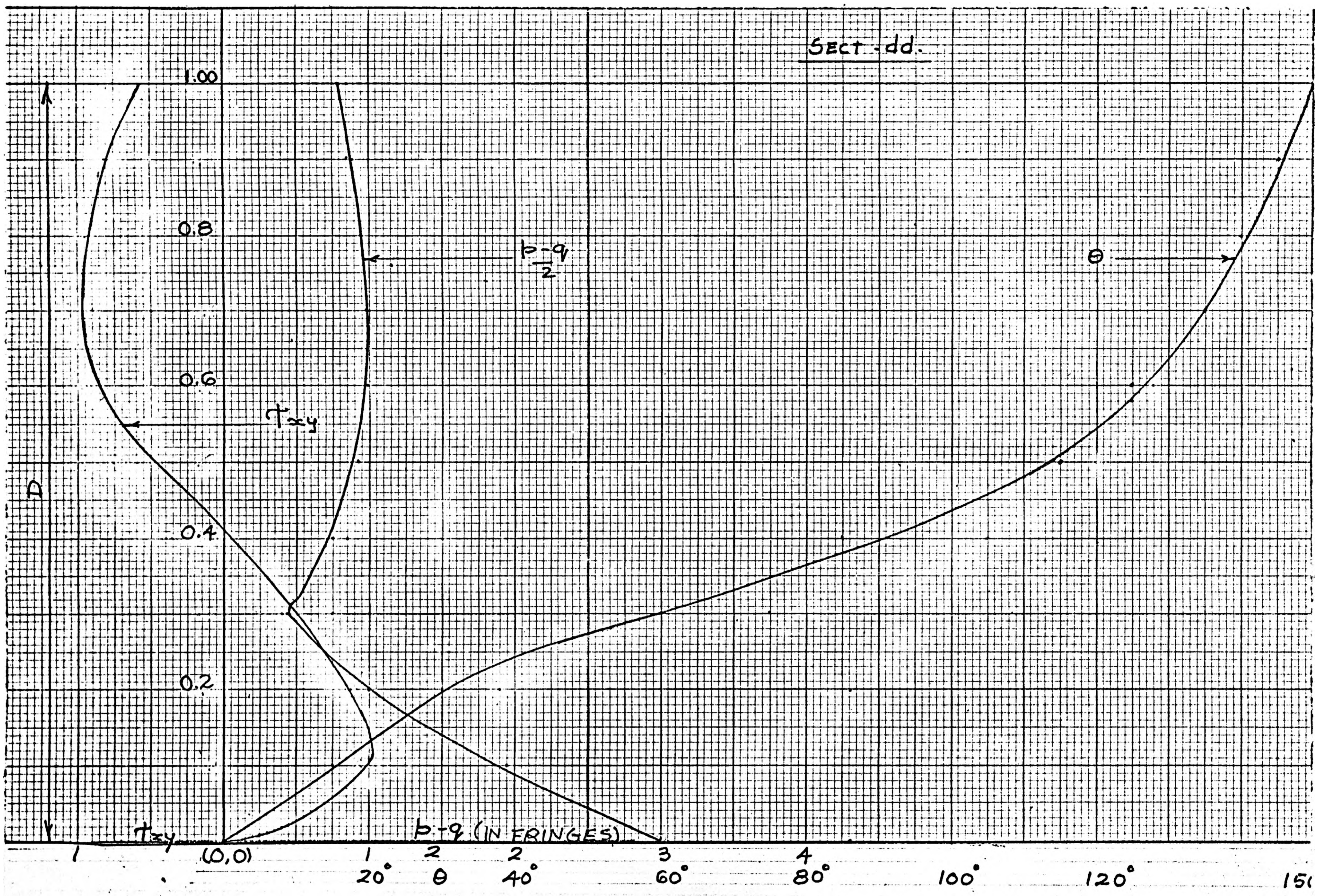
FRINGE PATTERN

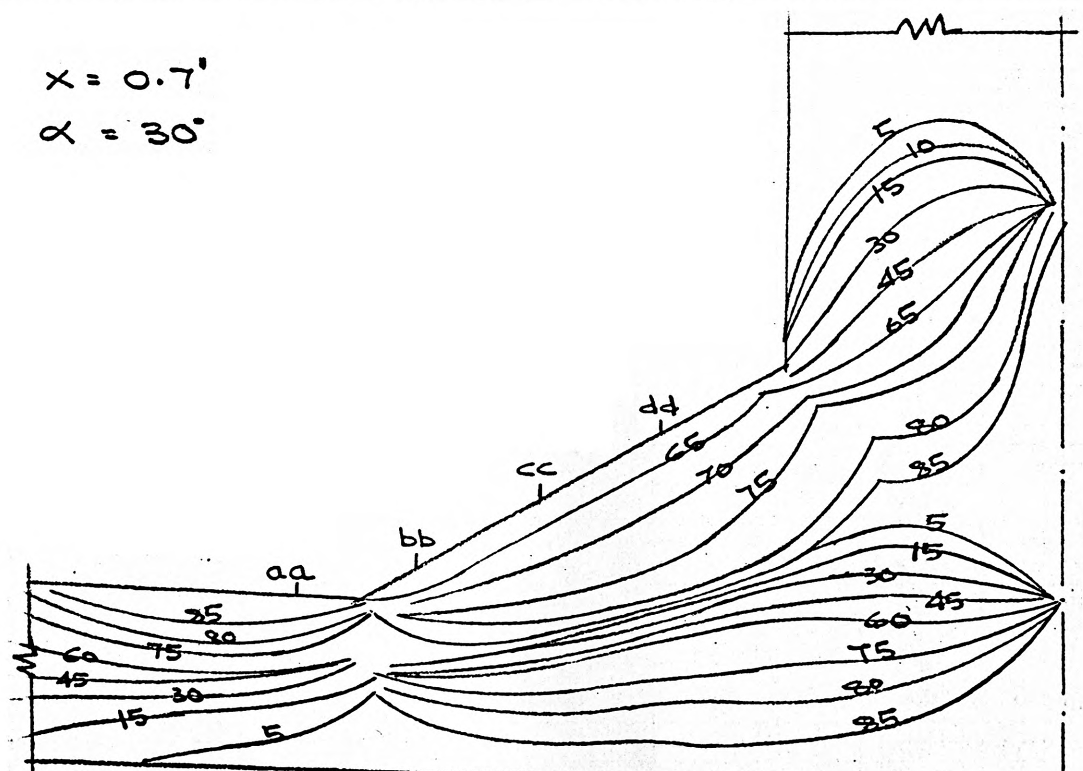
FIG. 12


SECT. aa
1.00
0.8
0.6
0.4
0.2
p-q/2
θ
τ_{xy}
τ_{xy}
p-q (IN FRINGES)
(0,0)
2
1
1
2
3
4
5
6
θ
20°
40°
60°
80°
100°

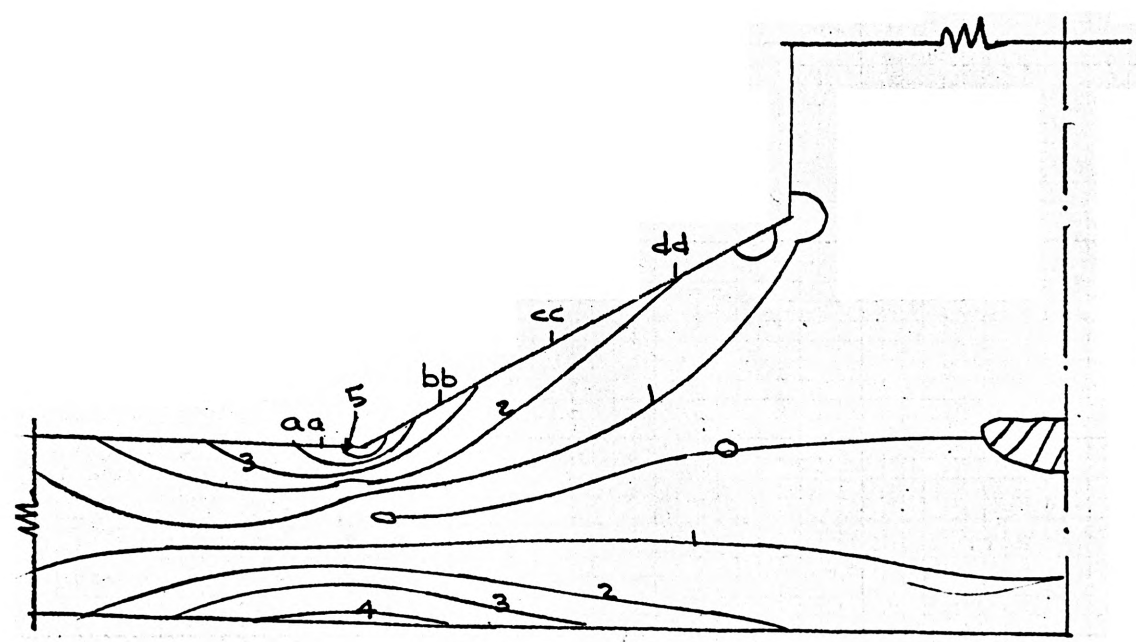

SECT bb
1.00
0.8
0.6
0.4
0.2
CORRECTED Txy
$\frac{p-q}{2}$
θ
A
3
2
1
Txy
(0,0)
2
1
2
3
4
5
p-q (IN FRINGES)
θ
0
20°
40°
60°
80°
100°
120°
150°


SECT CC
1.00
0.8
0.6
0.4
0.2
$\frac{p-q}{2}$
θ
τ_{xy}
A
τ_{xy}
p-q (IN FRINGES)
(0,0)
1
1
2
2
3
4
5
20°
θ
40°
60°
80°
100°
120°
150°


SECT. dd.
1.00
0.8
0.6
0.4
0.2
p-q/2
θ
τxy
D
τxy
(0,0)
p-q (IN FRINGES)
1
1
2
2
3
4
20°
θ
40°
60°
80°
100°
120°
150

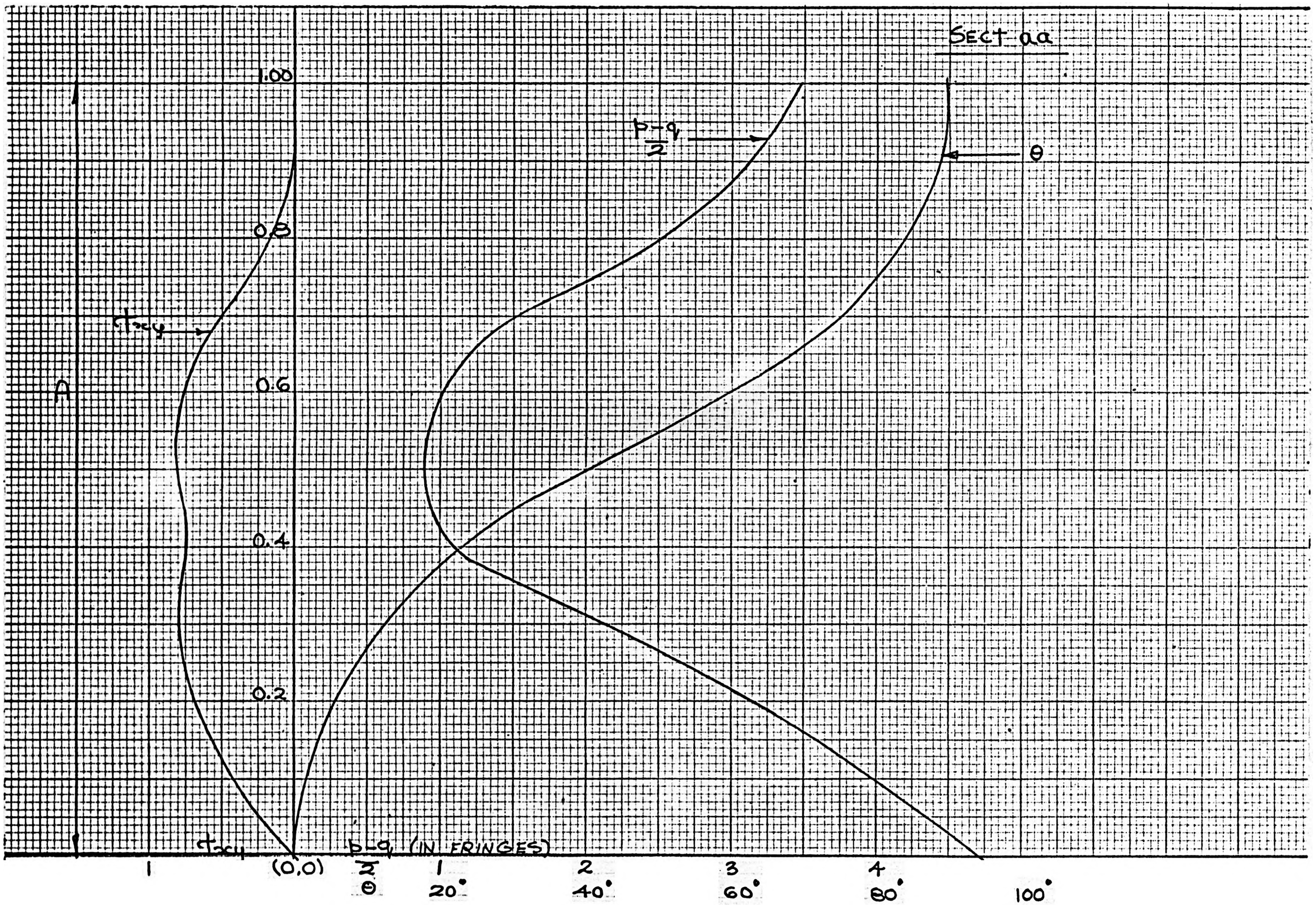


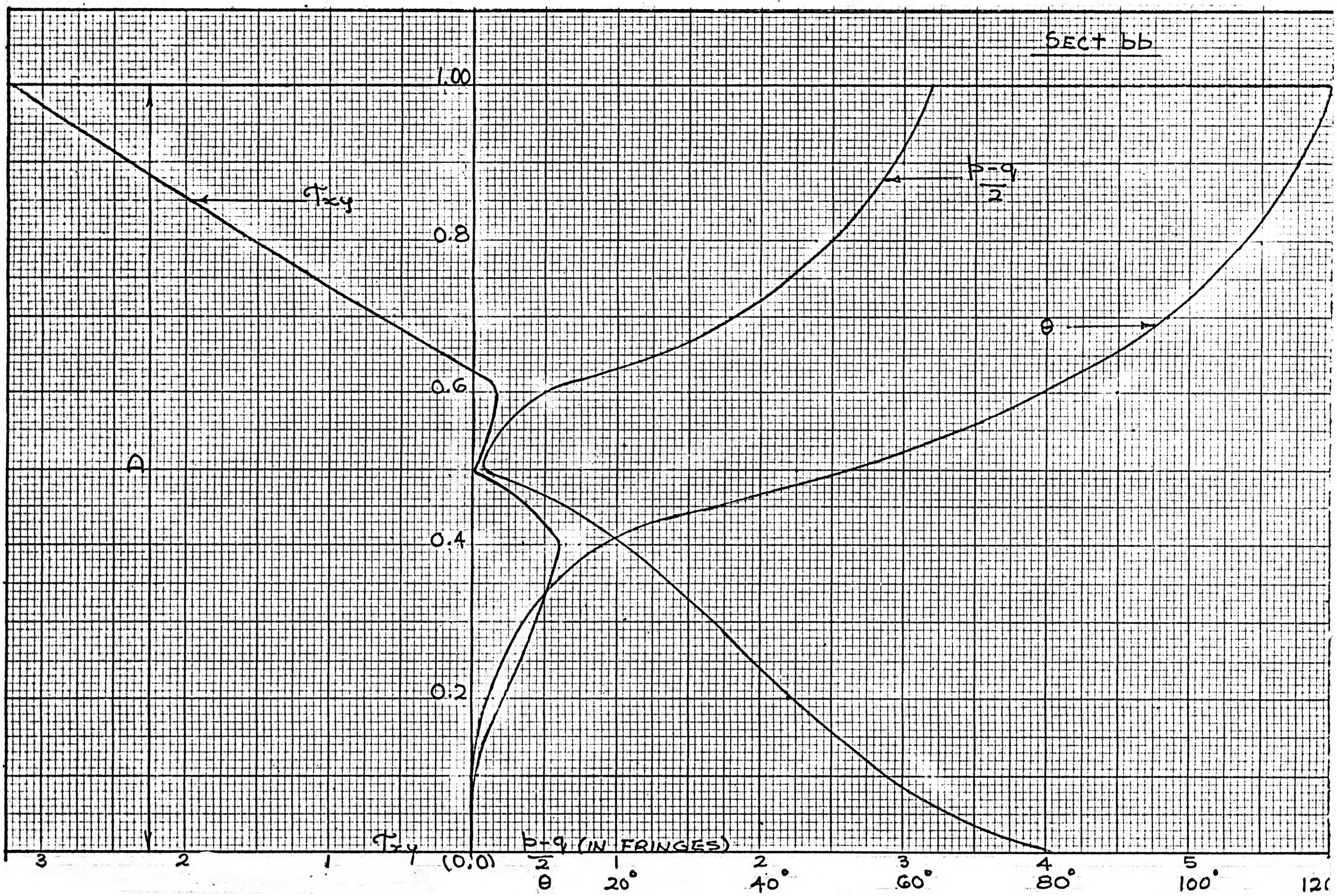
ISOCLINICS

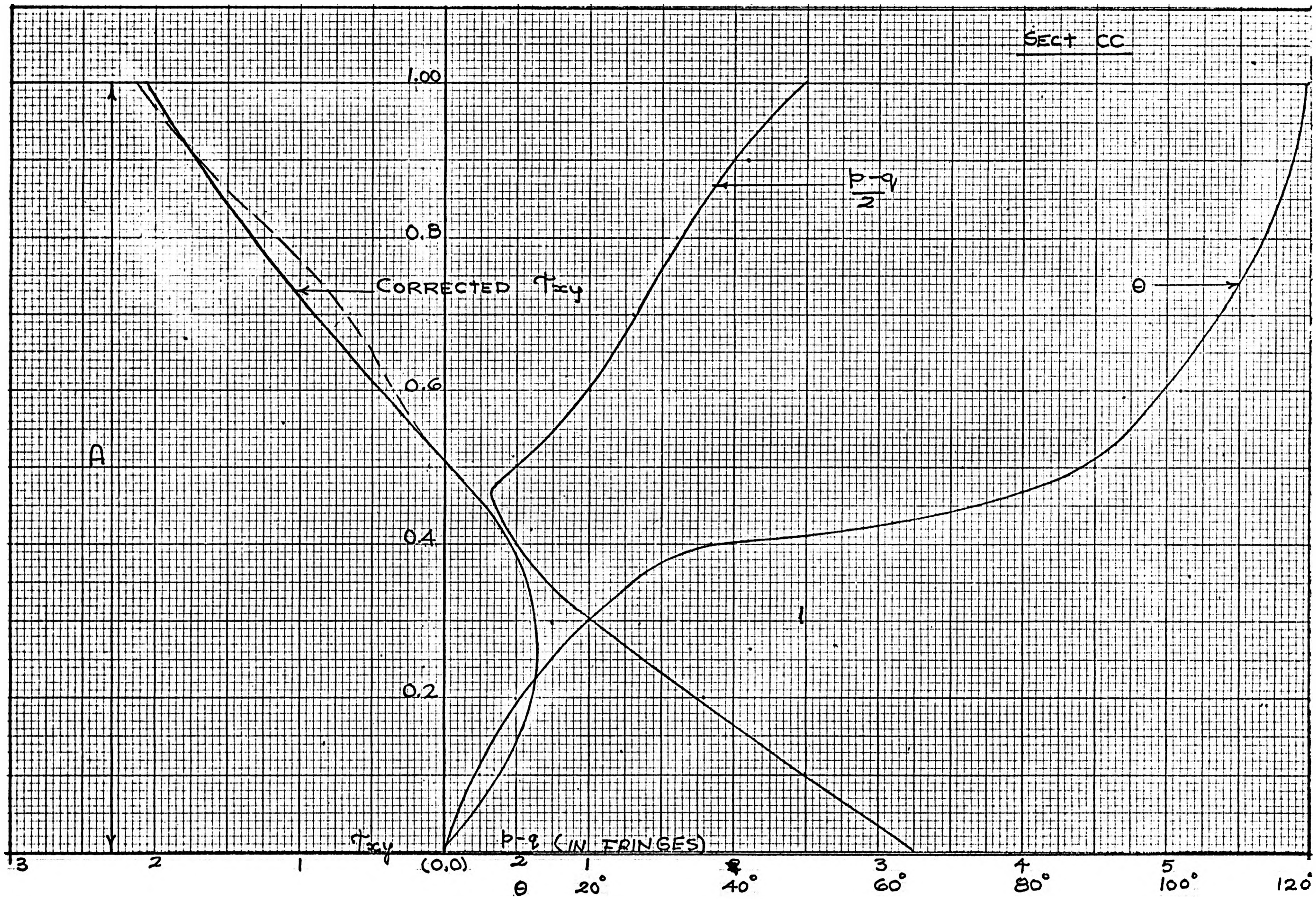


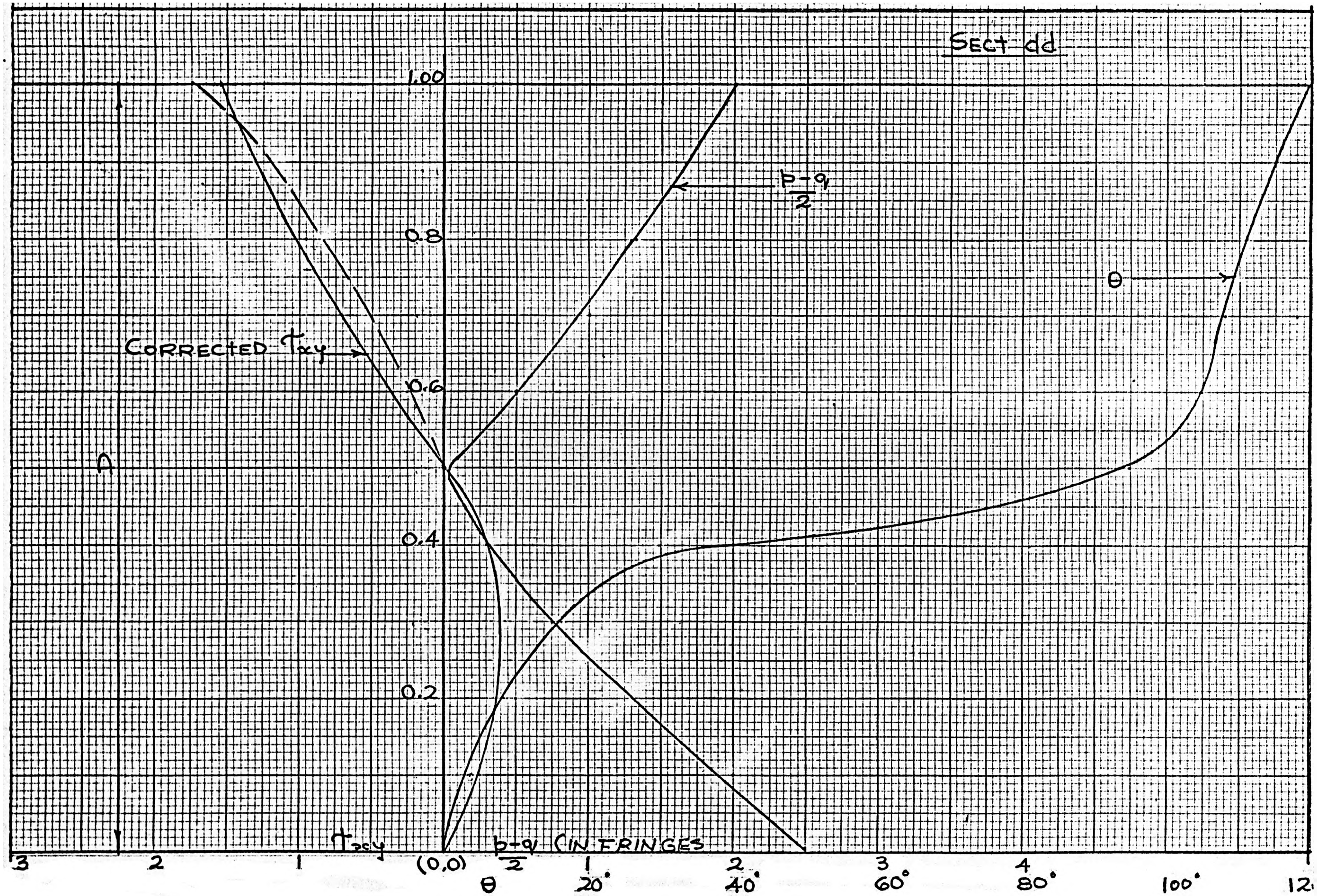
FRINGE PATTERN

FIG. 12

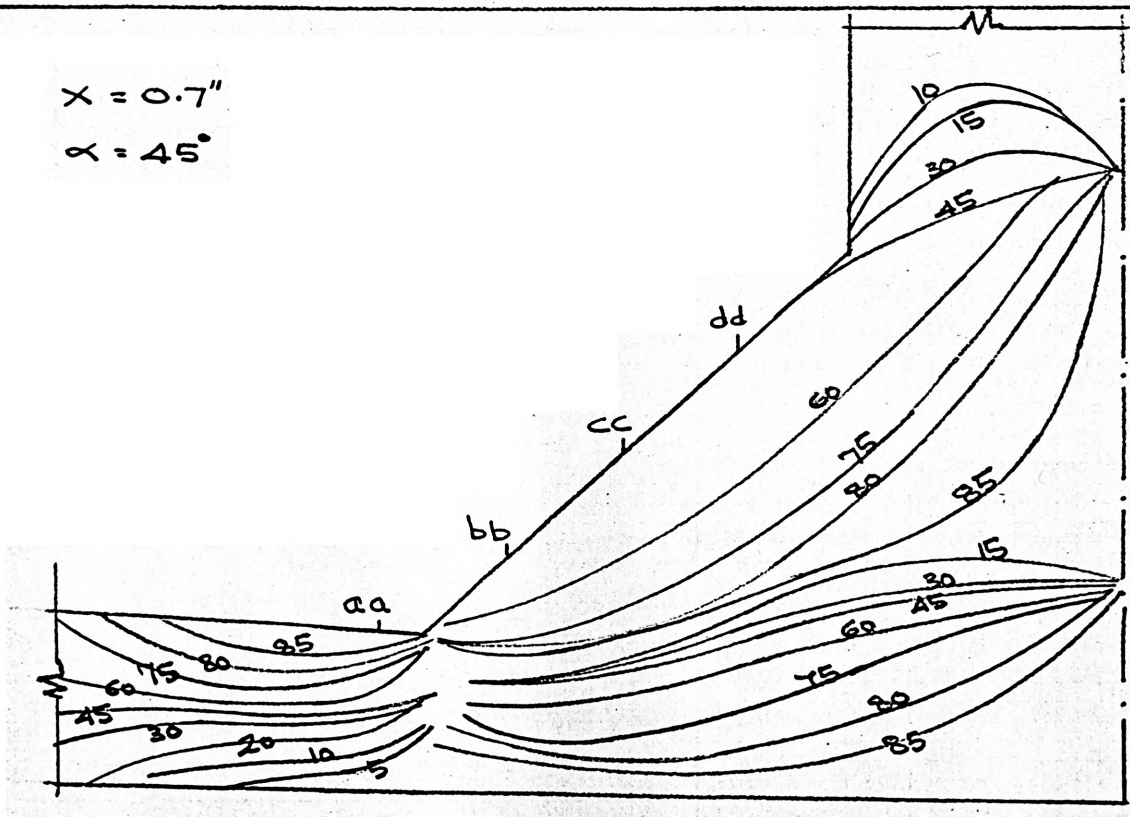

SECT aa
1.00
0.8
0.6
0.4
0.2
A
τ_{xy}
$\frac{p-q}{2}$
θ
τ_{xy}
(0,0)
$\frac{p-q}{2}$ (IN FRINGES)
θ
1
1
2
3
4
20°
40°
60°
80°
100°


SECT bb
1.00
0.8
0.6
0.4
0.2
τ_{xy}
$\frac{p-q}{2}$
θ
A
3
2
1
1
τ_{xy}
(0,0)
$\frac{p-q}{2}$ (IN FRINGES)
θ
1
20°
2
40°
3
60°
4
80°
5
100°
12(

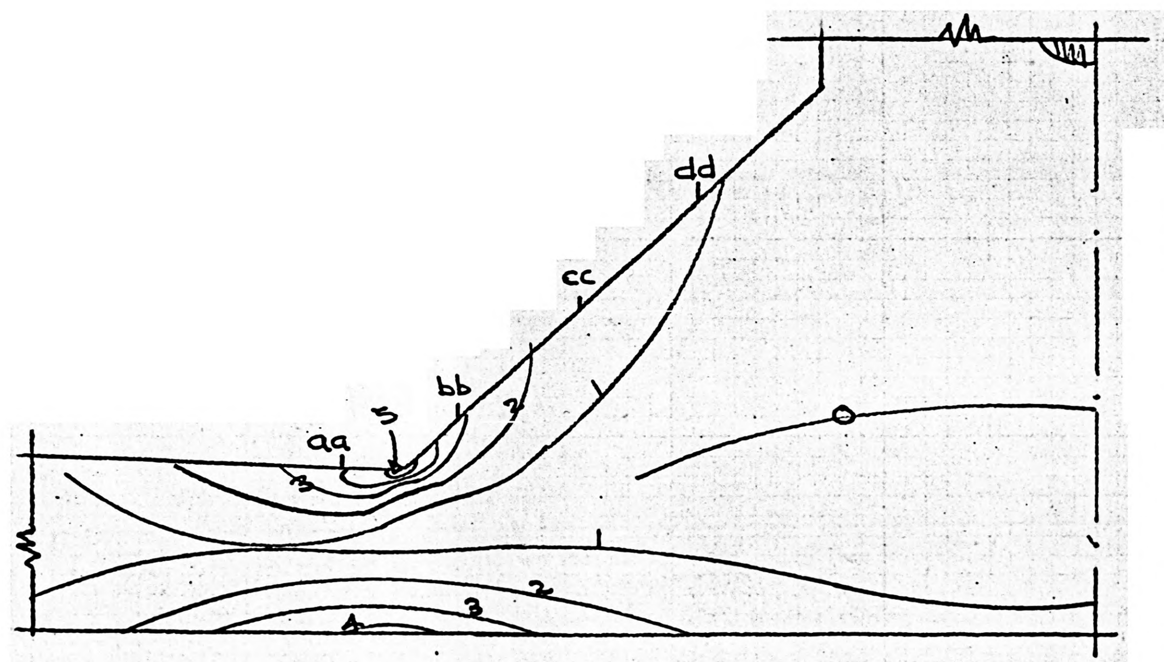

SECT CC
1.00
0.8
0.6
0.4
0.2
$\frac{p-q}{2}$
CORRECTED τ_{xy}
θ
A
τ_{xy}
p-q (IN FRINGES)
3
2
1
(0,0)
2
1
θ
20°
40°
3
60°
4
80°
5
100°
120°



Sect dd
1.00
0.8
0.6
0.4
0.2
$\frac{p-q}{2}$
θ
Corrected τ_{xy}
A
τ_{xy}
(0,0)
$\frac{p-q}{2}$ (IN FRINGES
θ
3
2
1
1
2
3
4
20°
40°
60°
80°
100°
12

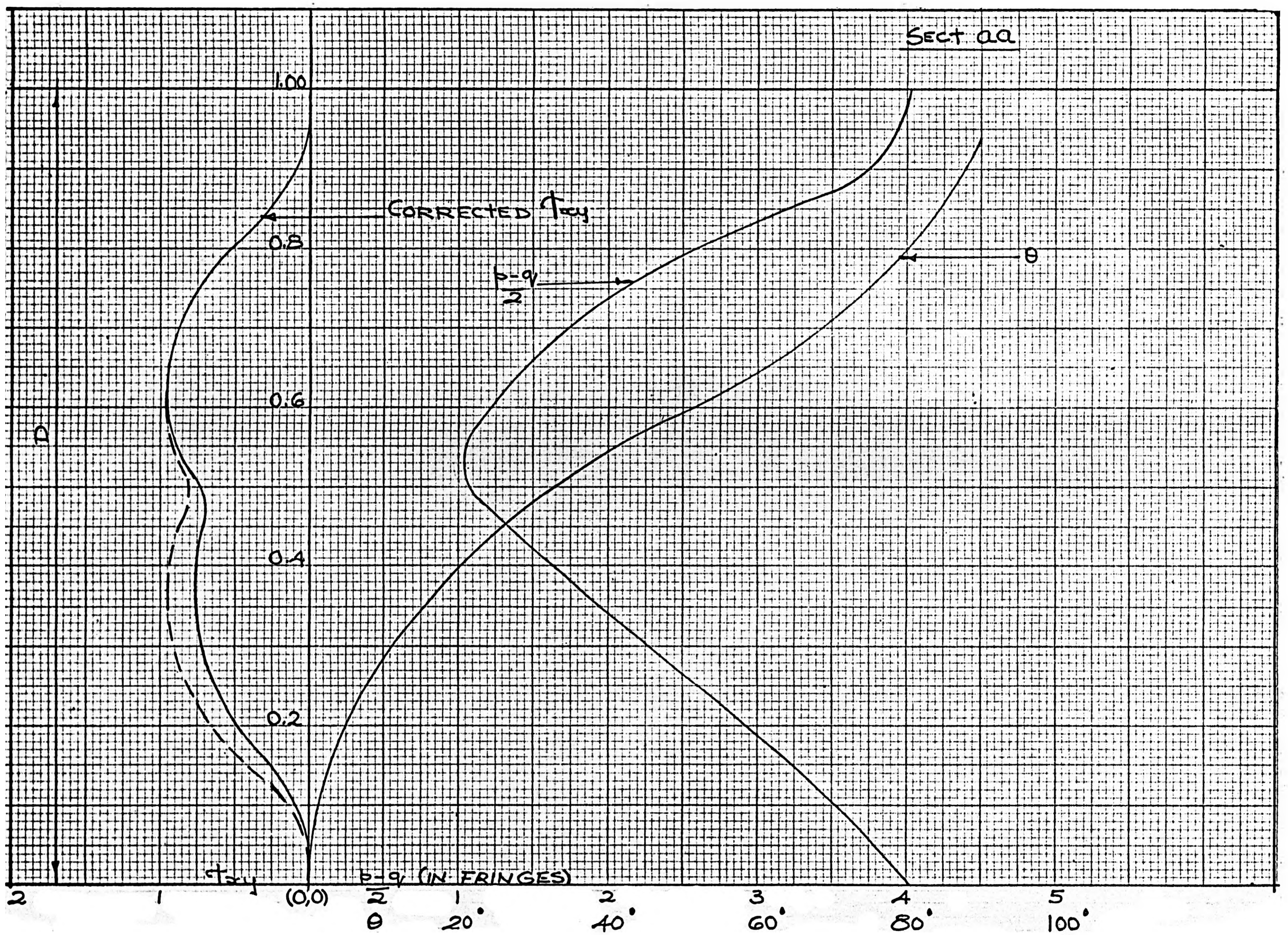


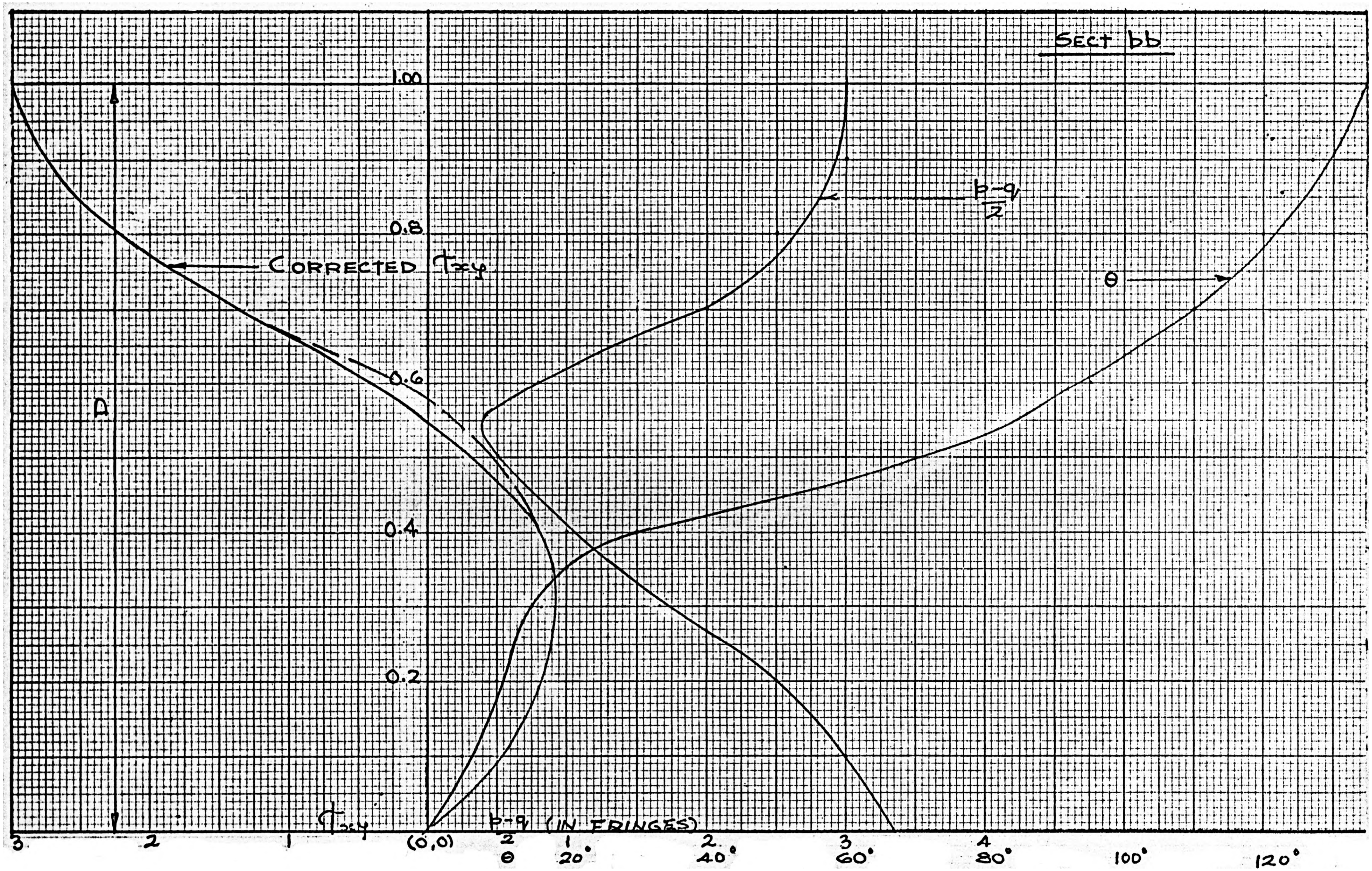
ISOCLINICS



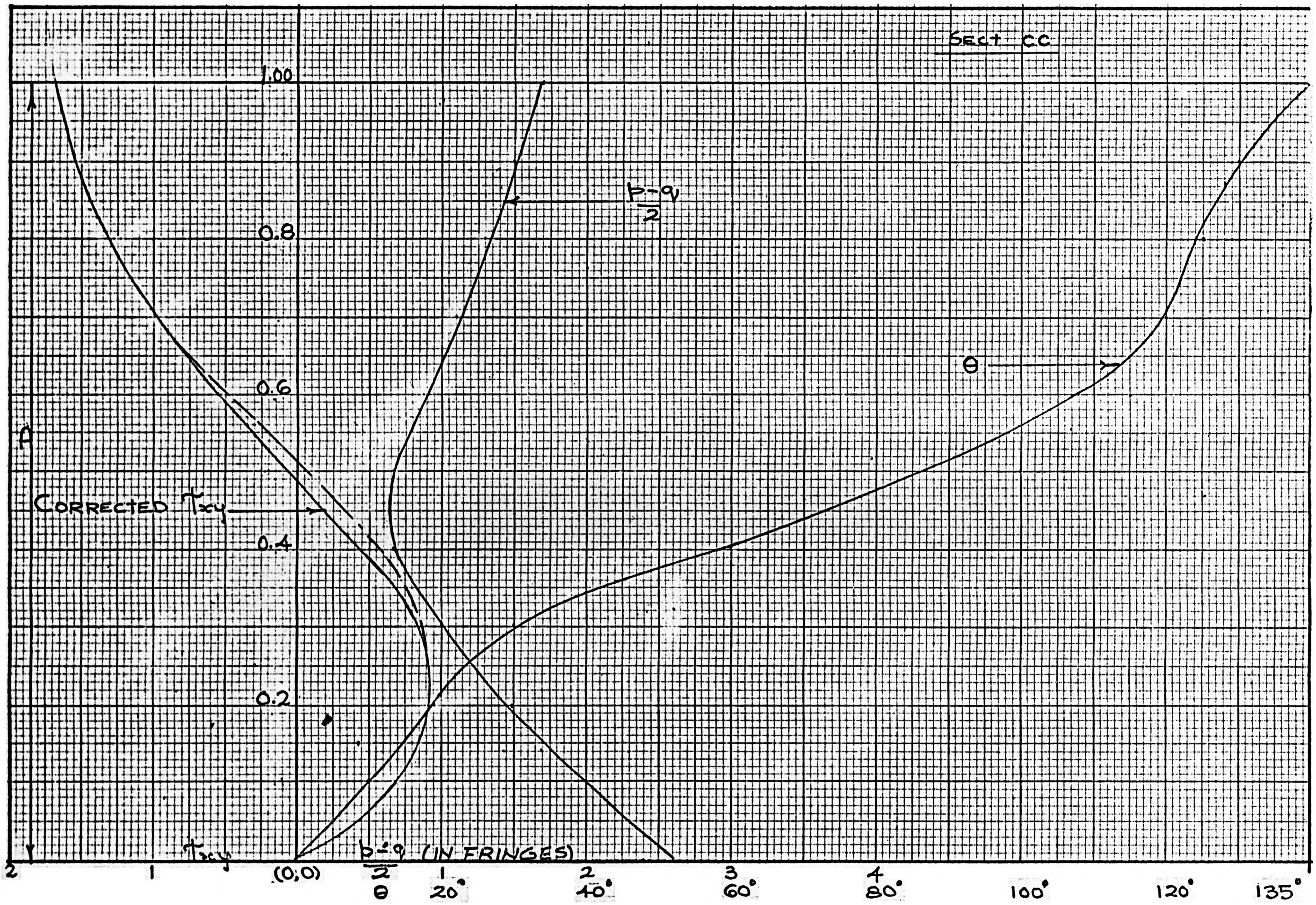
FRINGE PATTERN

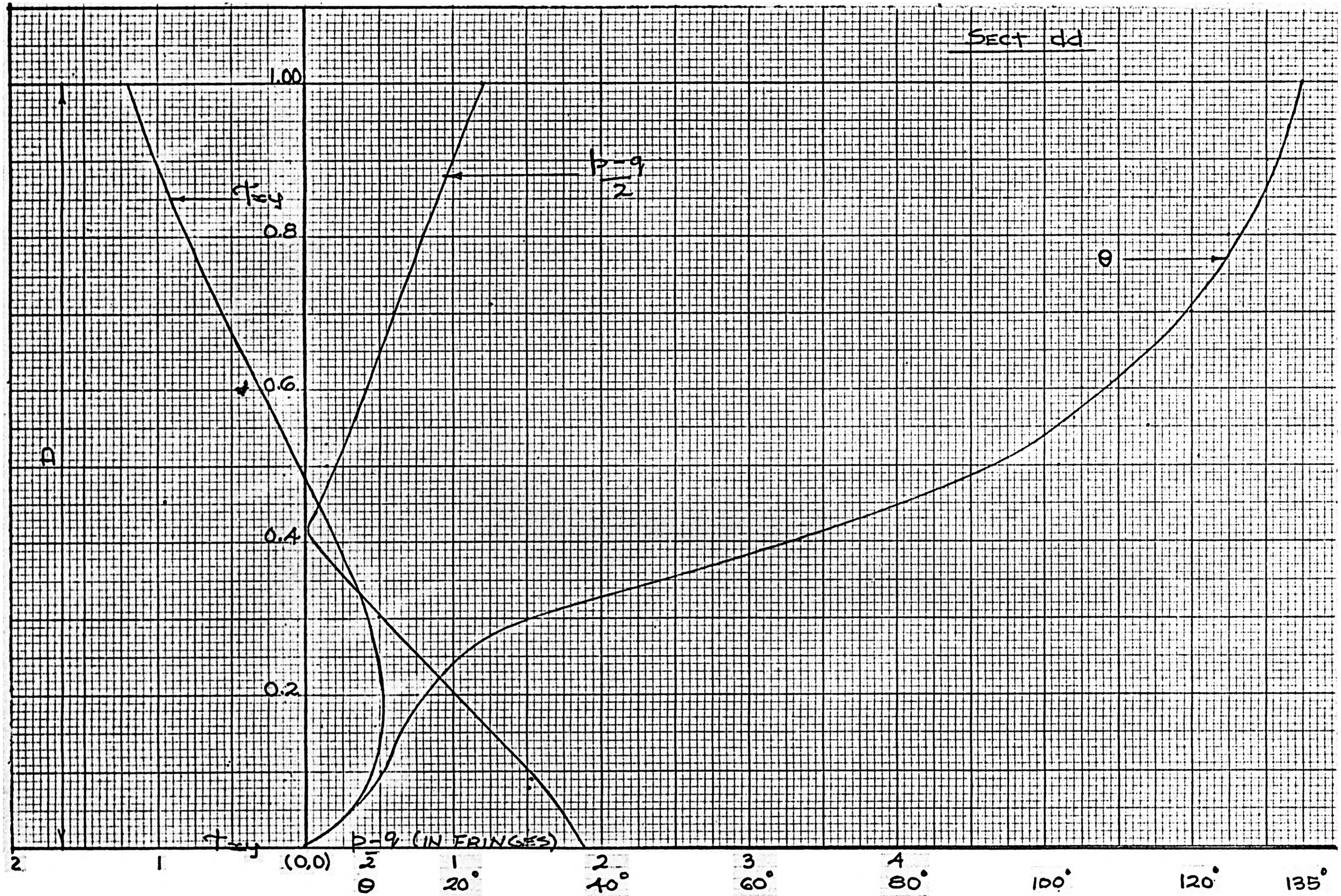
FIG. 12


Sect aa
1.00
0.8
0.6
0.4
0.2
D
CORRECTED Txy
p-q / 2
θ
Txy
(0,0)
p-q / 2 (IN FRINGES)
θ
2
1
1
2
3
4
5
20°
40°
60°
80°
100°



Sect bb
1.00
0.8
0.6
0.4
0.2
D
Corrected τ_{xy}
$\frac{p-q}{2}$
θ
τ_{xy}
p-q (IN FRINGES)
3
2
1
(0,0)
2
1
2
3
4
θ
20°
40°
60°
80°
100°
120°


Sect CC
1.00
0.8
0.6
0.4
0.2
A
Corrected τ_{xy}
$\frac{p-q}{2}$
θ
τ_{xy}
$\frac{p-q}{2}$ (IN FRINGES)
2
1
(0,0)
θ
1
2
3
4
20°
40°
60°
80°
100°
120°
135°


SECT dd
1.00
0.8
0.6
0.4
0.2
A
τ_{xy}
$\frac{p-q}{2}$
θ
2
1
(0,0)
$\frac{p-q}{2}$ (IN FRINGES)
1
2
3
4
θ
20°
40°
60°
80°
100°
120°
135°

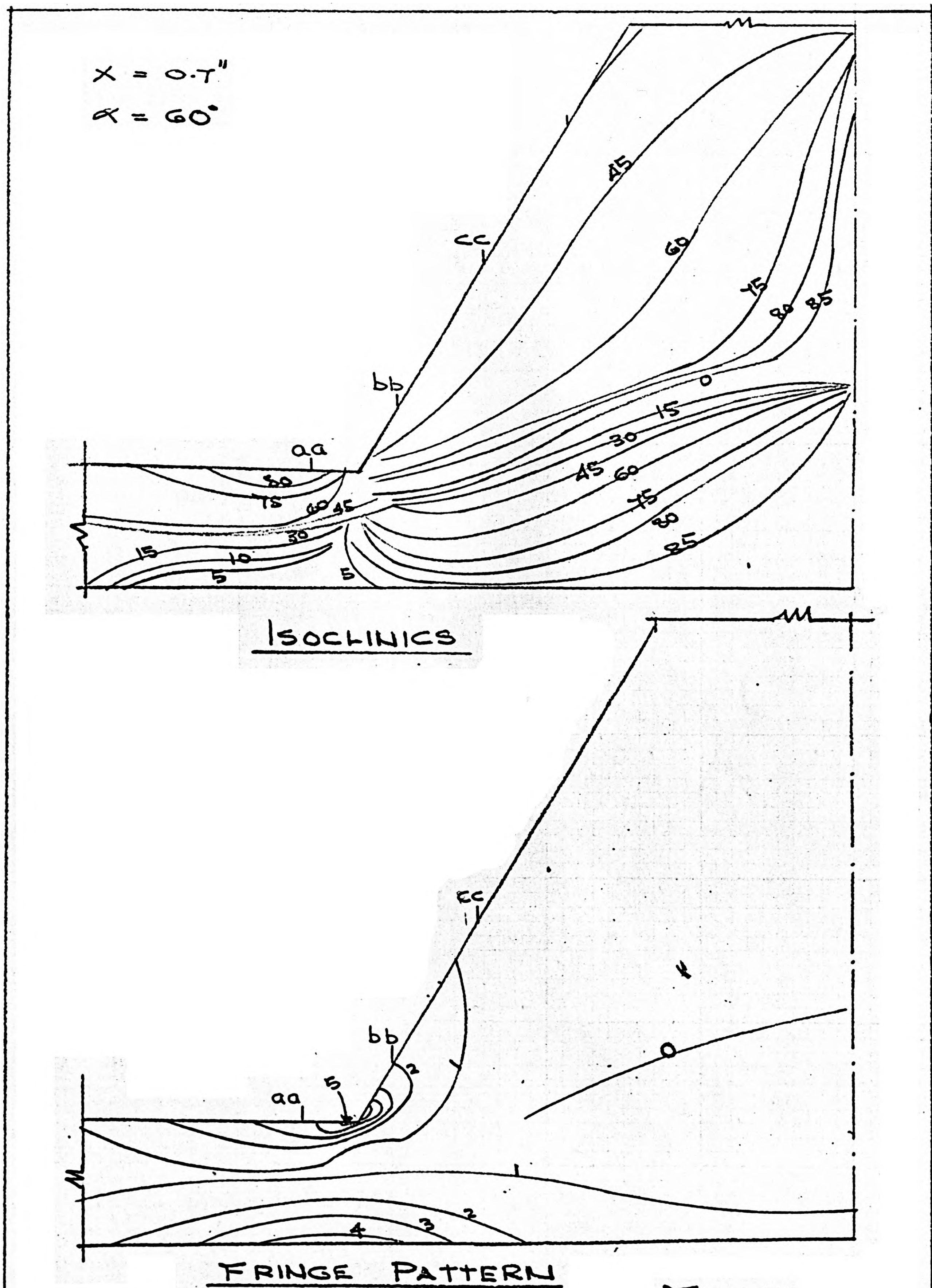
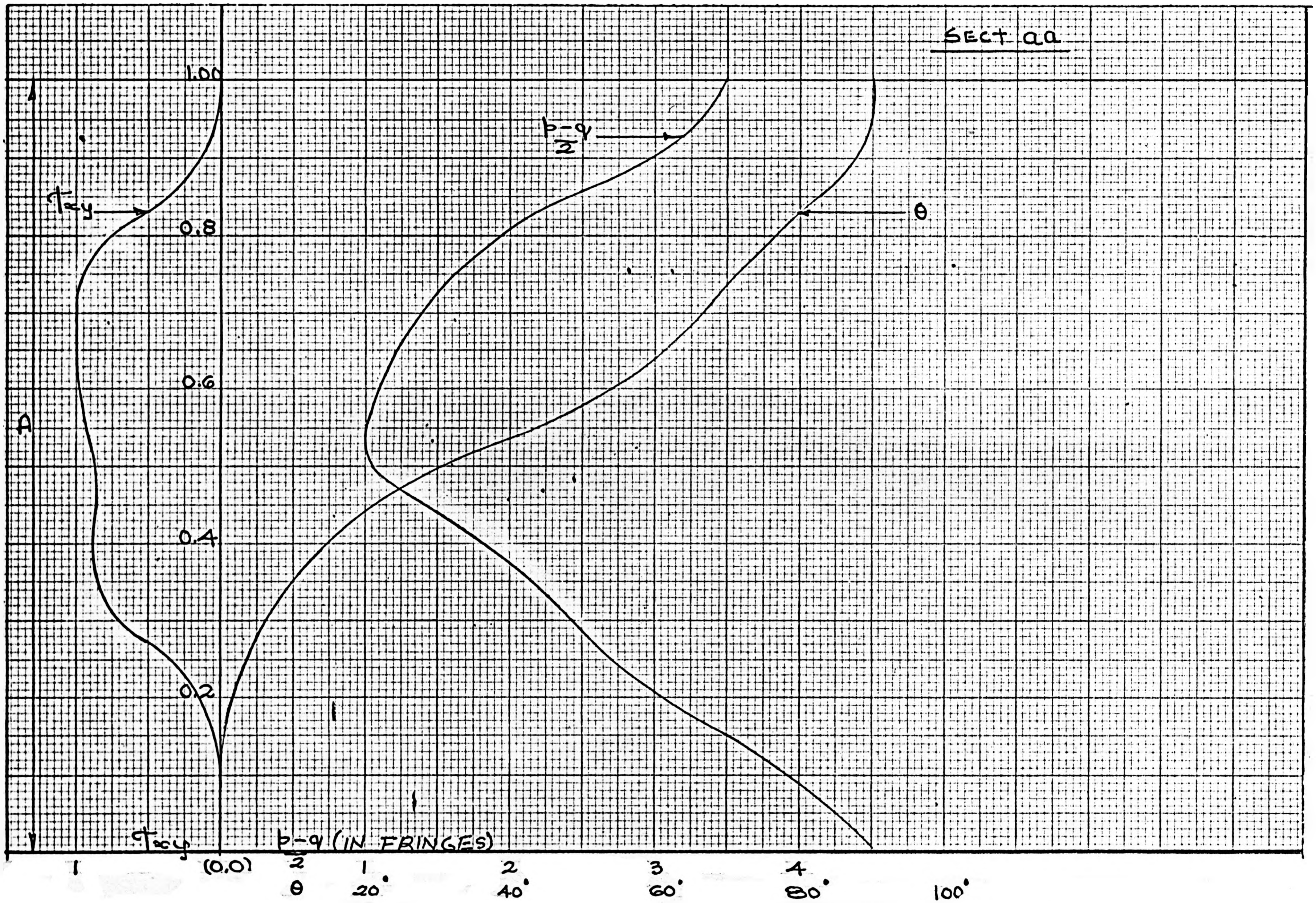
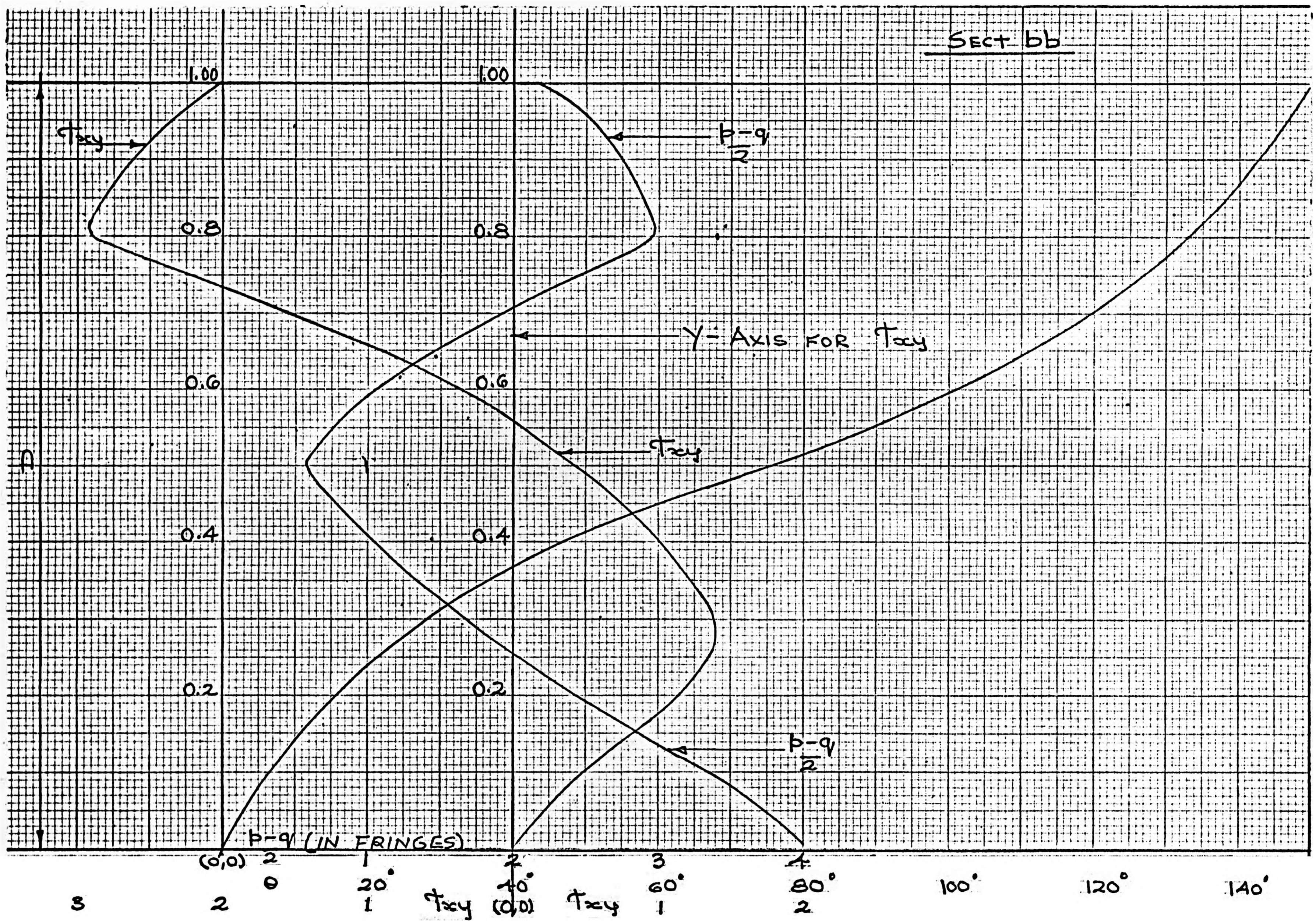
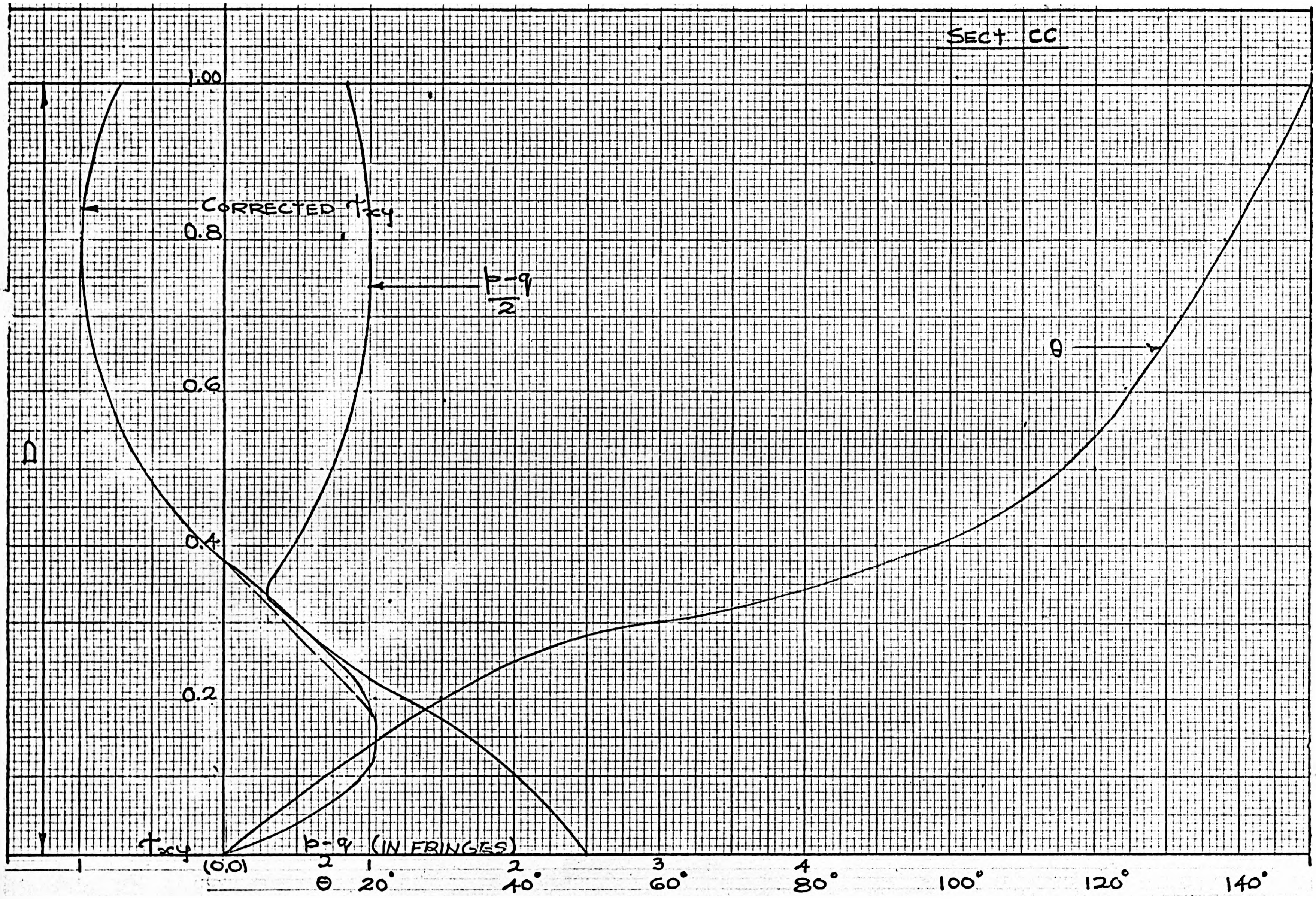
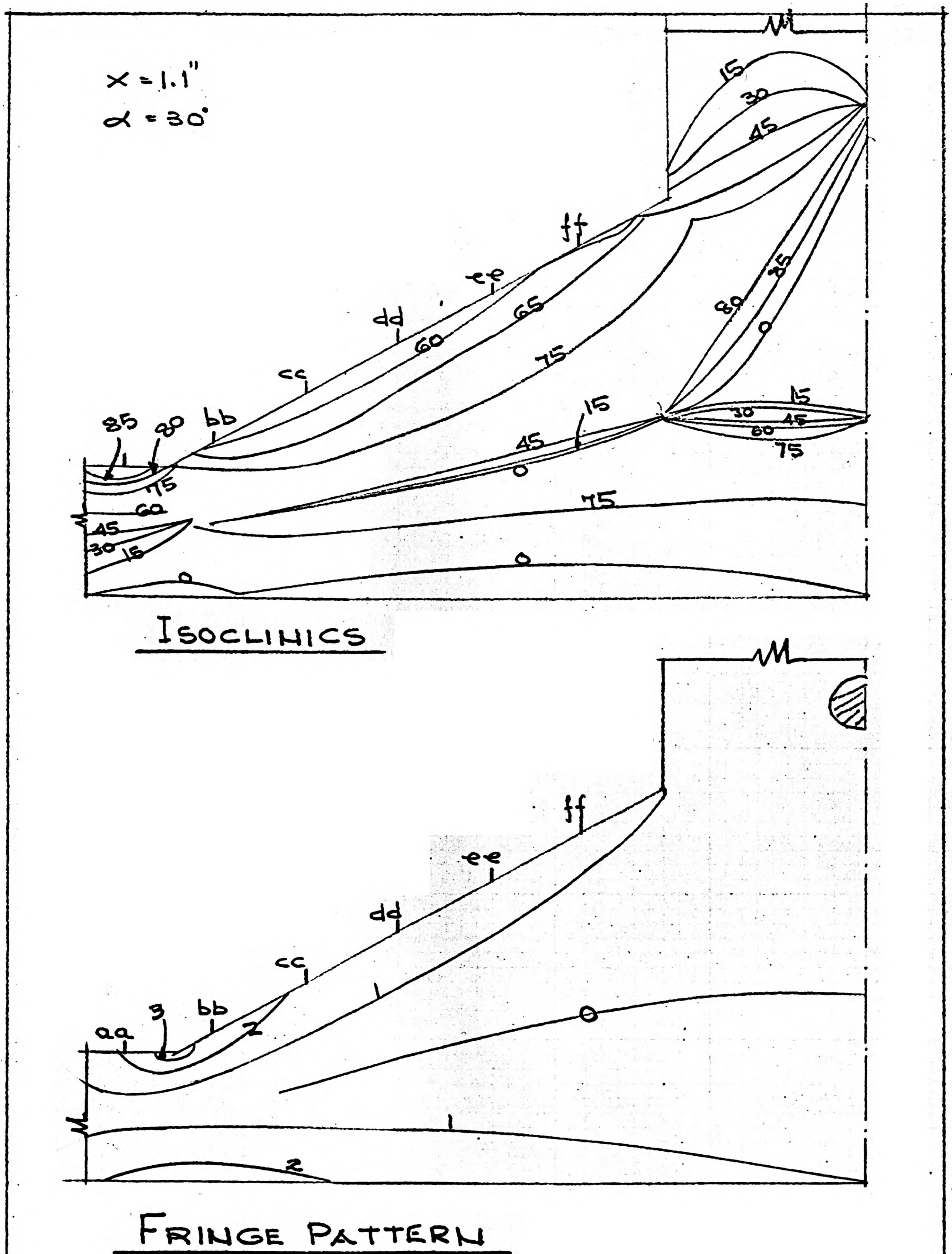

X = 0.7"
α = 60°
cc
bb
aa
45
60
75
80
85
0
15
30
10
5
Isoclinics
cc
bb
aa
5
2
1
0
4
3
2
Fringe Pattern

FIG. 12


SECT aa
1.00
0.8
0.6
0.4
0.2
$\frac{p-q}{2}$
τ_{xy}
θ
A
τ_{xy}
$\frac{p-q}{2}$ (IN FRINGES)
1
(0,0)
1
2
3
4
θ
20°
40°
60°
80°
100°

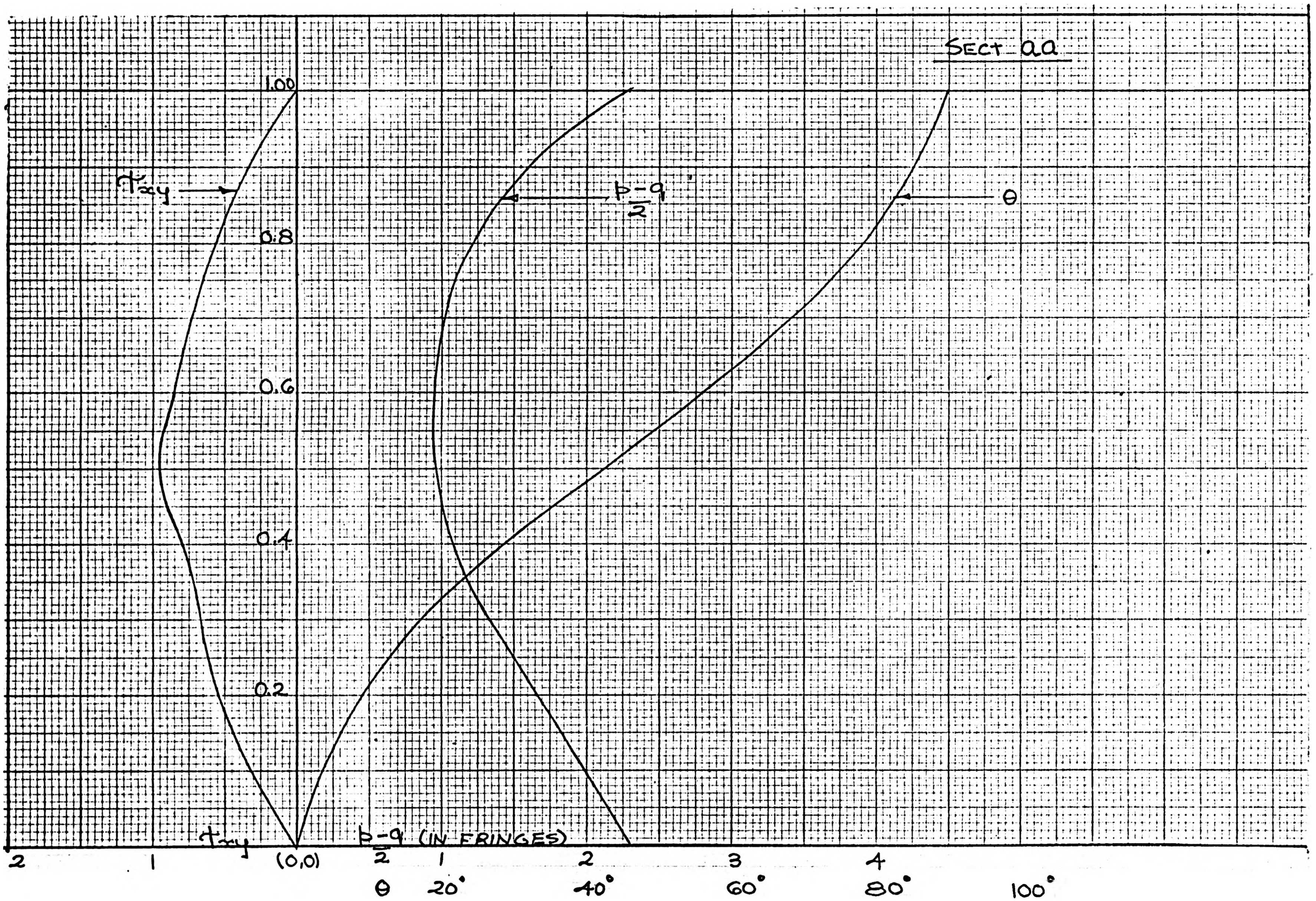

Sect bb
1.00
1.00
0.8
0.8
0.6
0.6
0.4
0.4
0.2
0.2
τxy
p-q/2
Y-Axis for τxy
τxy
p-q/2
A
p-q (IN FRINGES)
(0,0) 2 1 2 3 4
0 20° 40° 60° 80° 100° 120° 140°
3 2 1 τxy (0,0) τxy 1 2

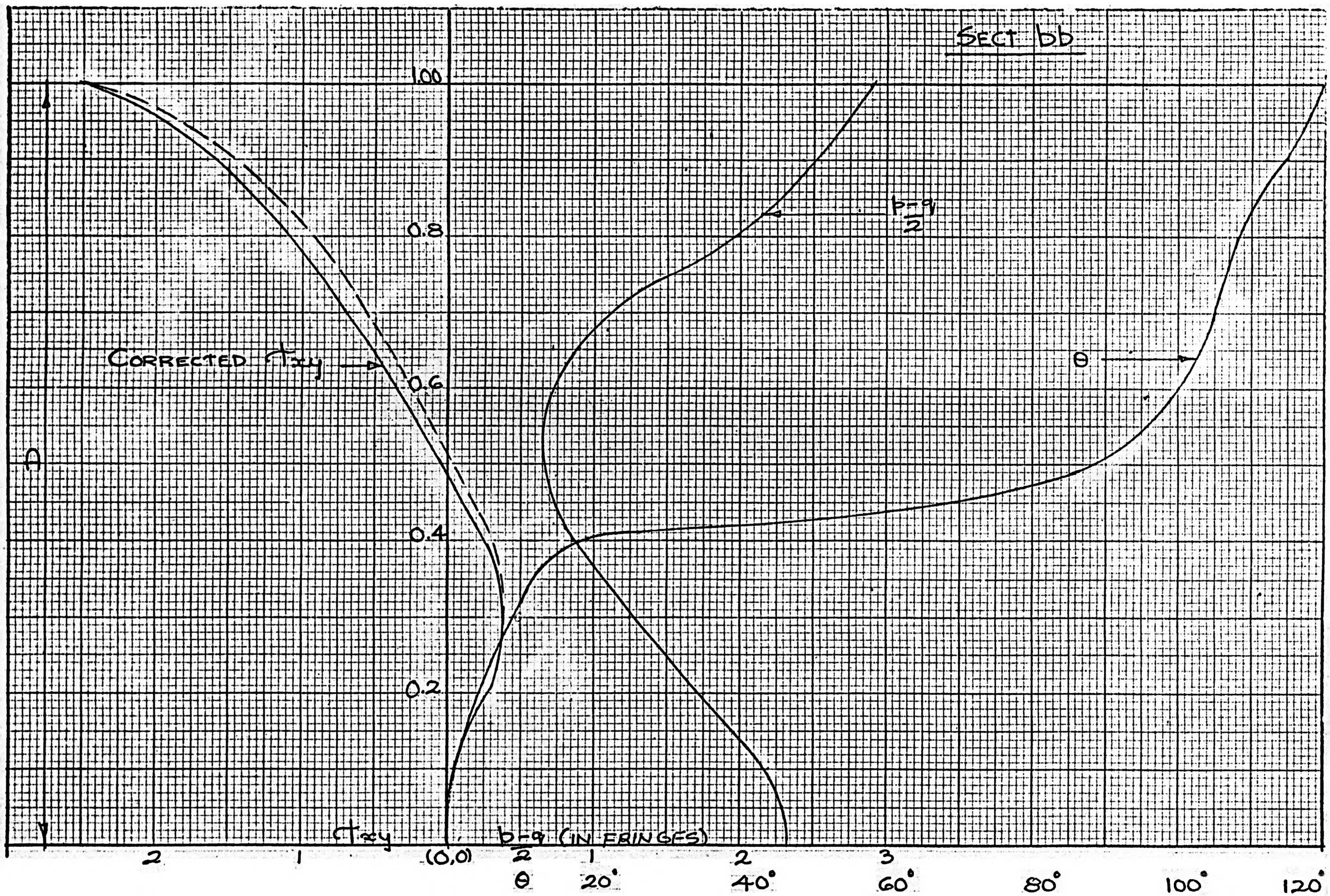

SECT CC
1.00
CORRECTED τ_{xy}
0.8
$\frac{p-q}{2}$
0.6
θ
D
0.4
0.2
τ_{xy}
p-q (IN FRINGES)
1
(0,0)
2
1
2
3
4
θ
20°
40°
60°
80°
100°
120°
140°

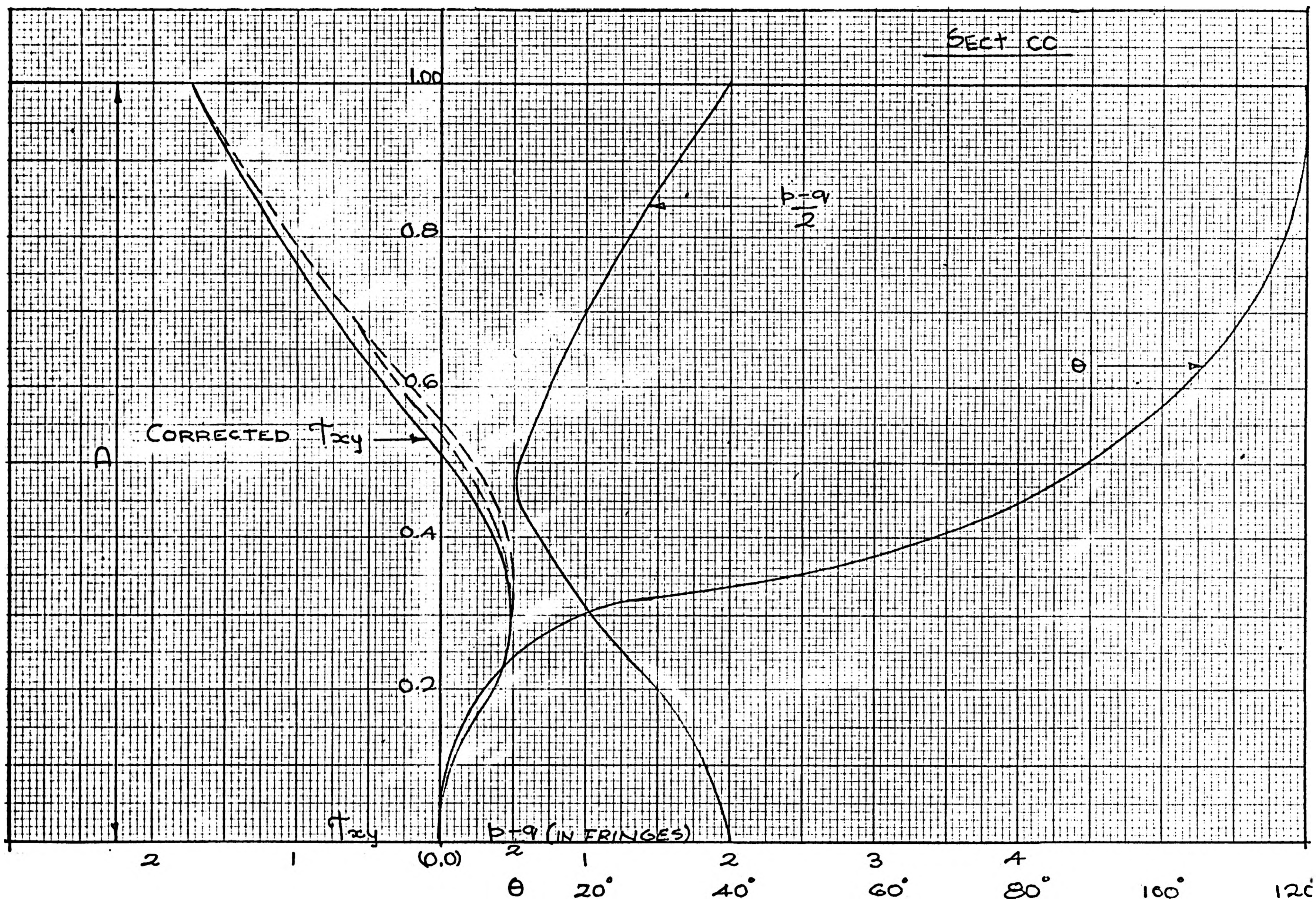


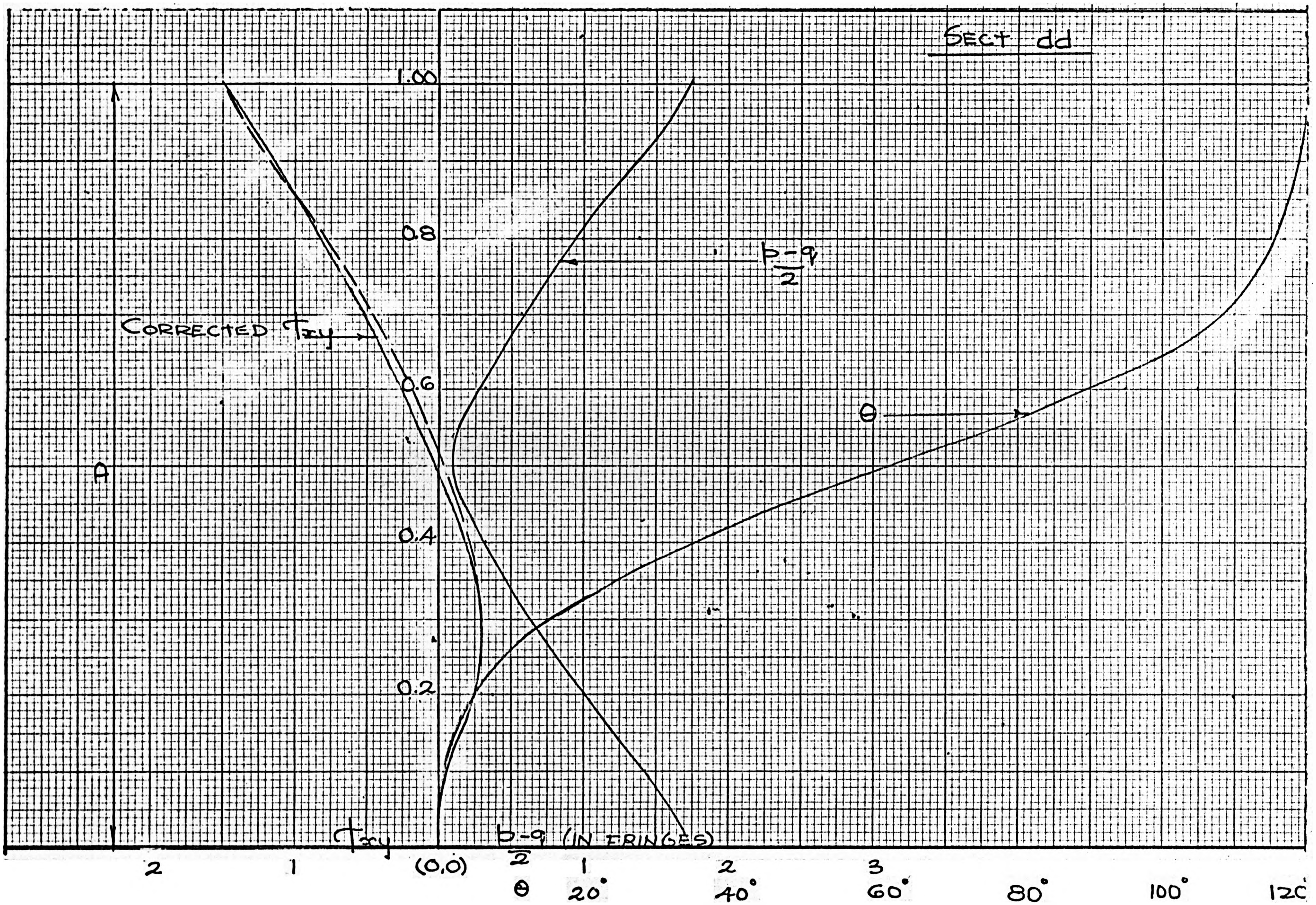
x = 1.1"
α = 30°
15
30
45
ff
ee
dd
cc
bb
85
80
65
60
75
35
80
0
15
45
30
60
75
Isoclinics
ff
ee
dd
cc
bb
aa
3
2
1
0
Fringe Pattern

FIG. 12

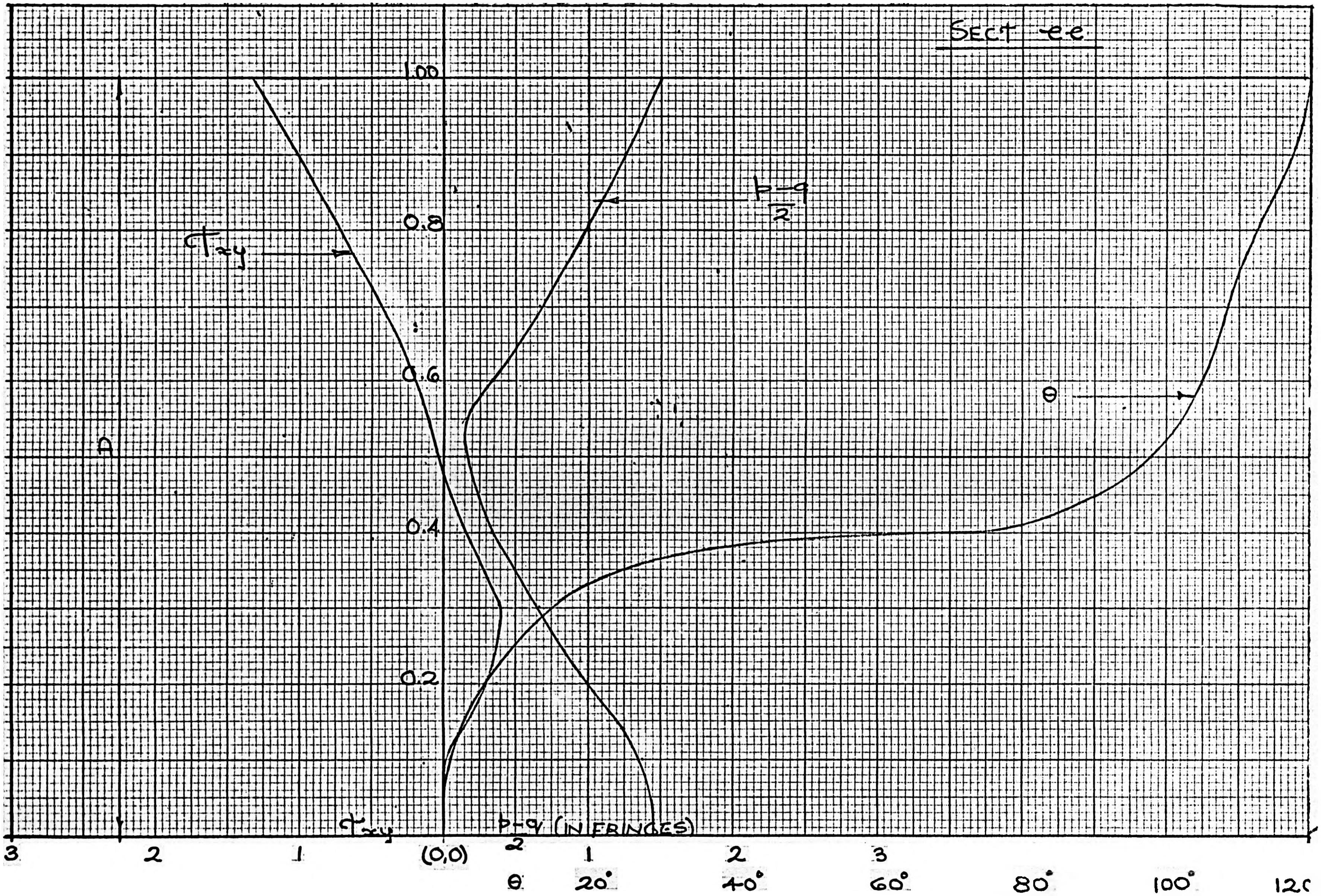

SECT aa
1.00
0.8
0.6
0.4
0.2
τ_{xy}
$\frac{p-q}{2}$
θ
τ_{xy}
(0,0)
$\frac{p-q}{2}$ (IN FRINGES)
2
1
1
2
3
4
θ
20°
40°
60°
80°
100°


SECT bb
1.00
0.8
0.6
0.4
0.2
CORRECTED τxy
b-a/2
θ
A
τxy
(0,0)
b-a/2 (IN FRINGES)
θ
2
1
1
2
3
20°
40°
60°
80°
100°
120°

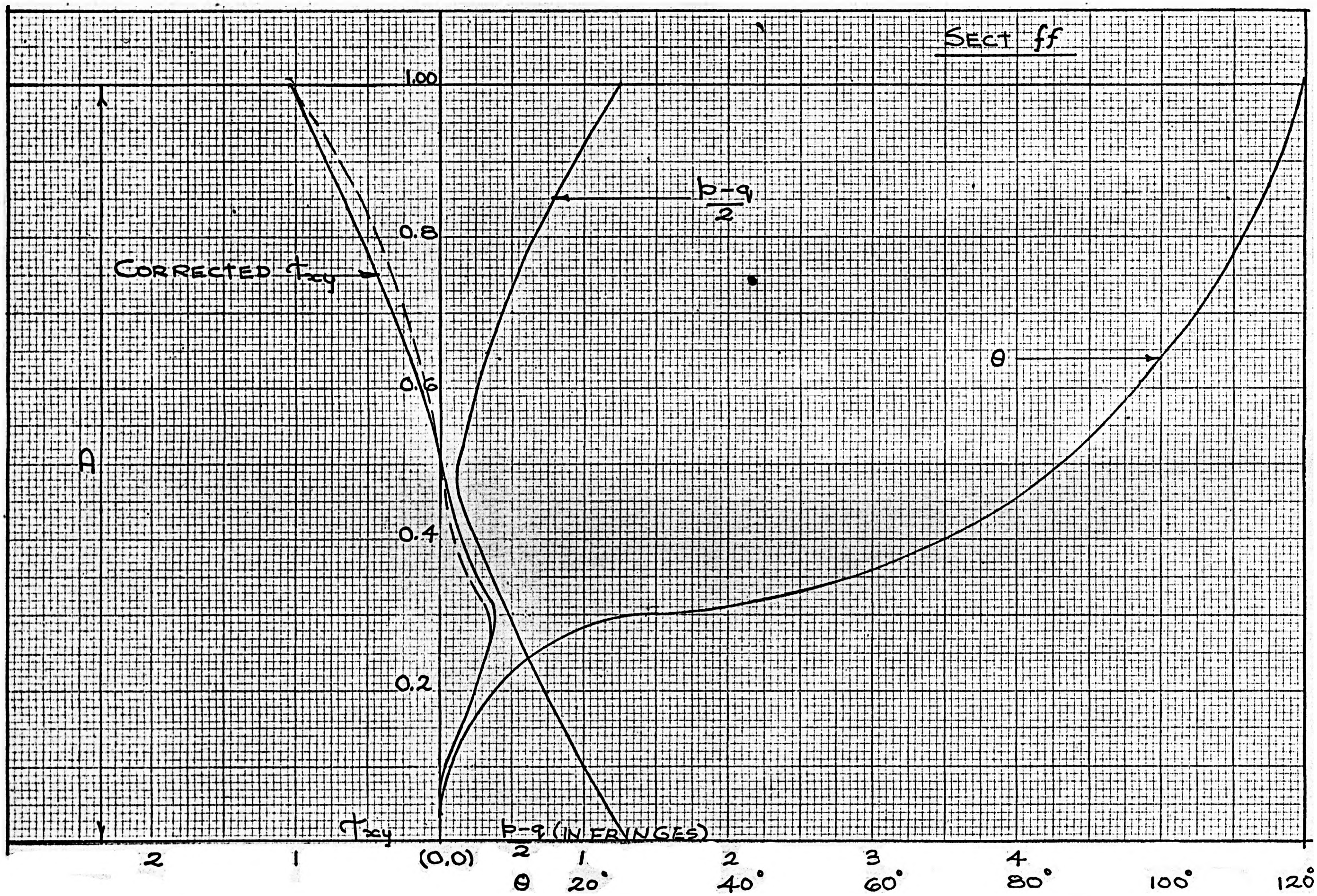

SECT CC
1.00
0.8
0.6
0.4
0.2
A
CORRECTED τxy
(p-q)/2
θ
τxy
p-q (IN FRINGES)
2
1
(0,0)
2
1
2
3
4
θ
20°
40°
60°
80°
100°
120

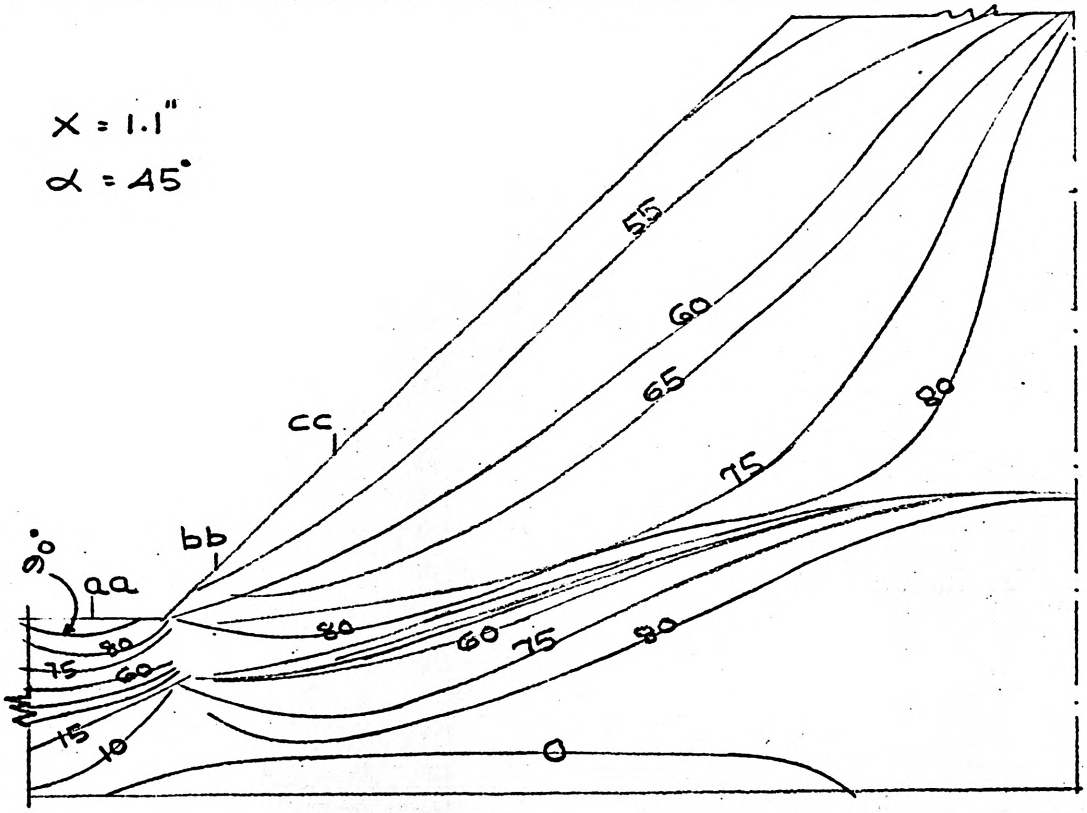


SECT dd
1.00
0.8
0.6
0.4
0.2
A
CORRECTED τxy
b-q / 2
θ
τxy
b-q / 2 (IN FRINGES)
2
1
(0,0)
1
2
3
θ
20°
40°
60°
80°
100°
120

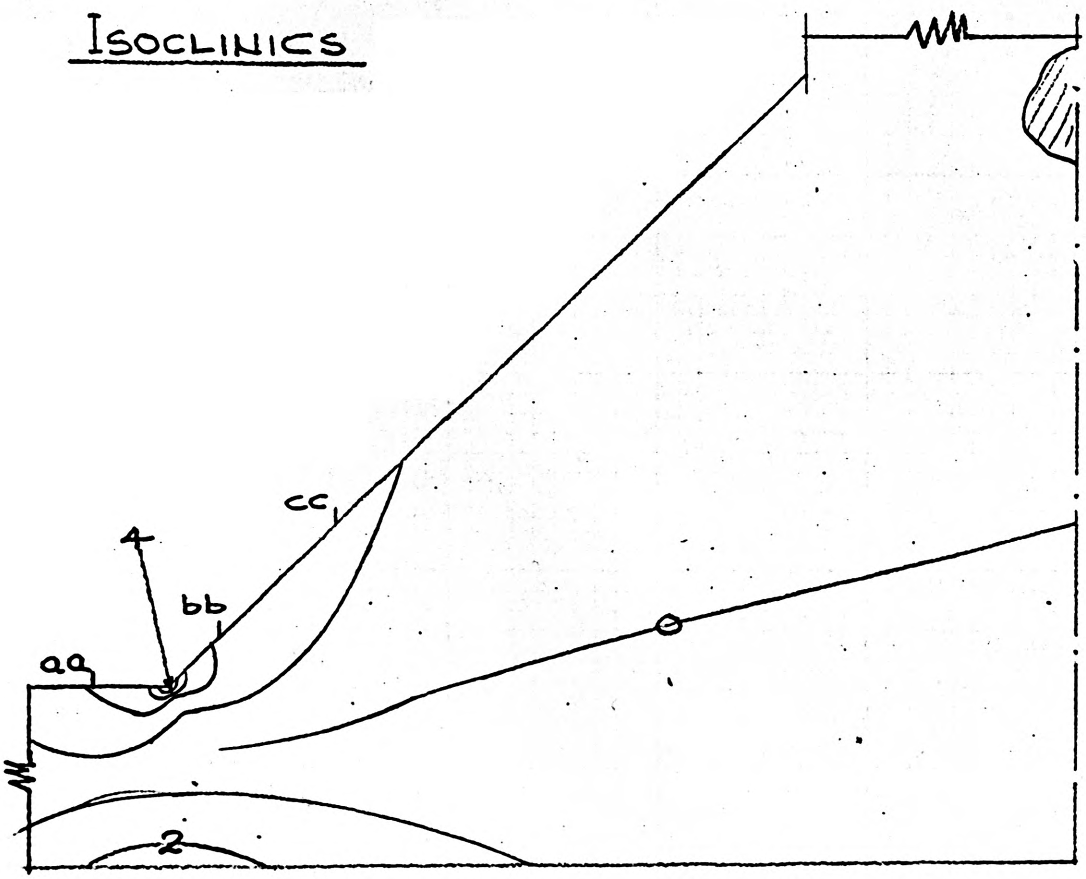


SECT ee
1.00
0.8
0.6
0.4
0.2
A
τ_{xy}
$\frac{p-q}{2}$
θ
τ_{xy}
$\frac{p-q}{2}$ (IN FRINGES)
3
2
1
(0,0)
1
2
3
θ
20°
40°
60°
80°
100°
120


SECT ff
1.00
0.8
0.6
0.4
0.2
CORRECTED τ_{xy}
$\frac{p-q}{2}$
θ
A
τ_{xy}
$\frac{p-q}{2}$ (IN FRINGES)
2
1
(0,0)
1
2
3
4
θ
20°
40°
60°
80°
100°
120°

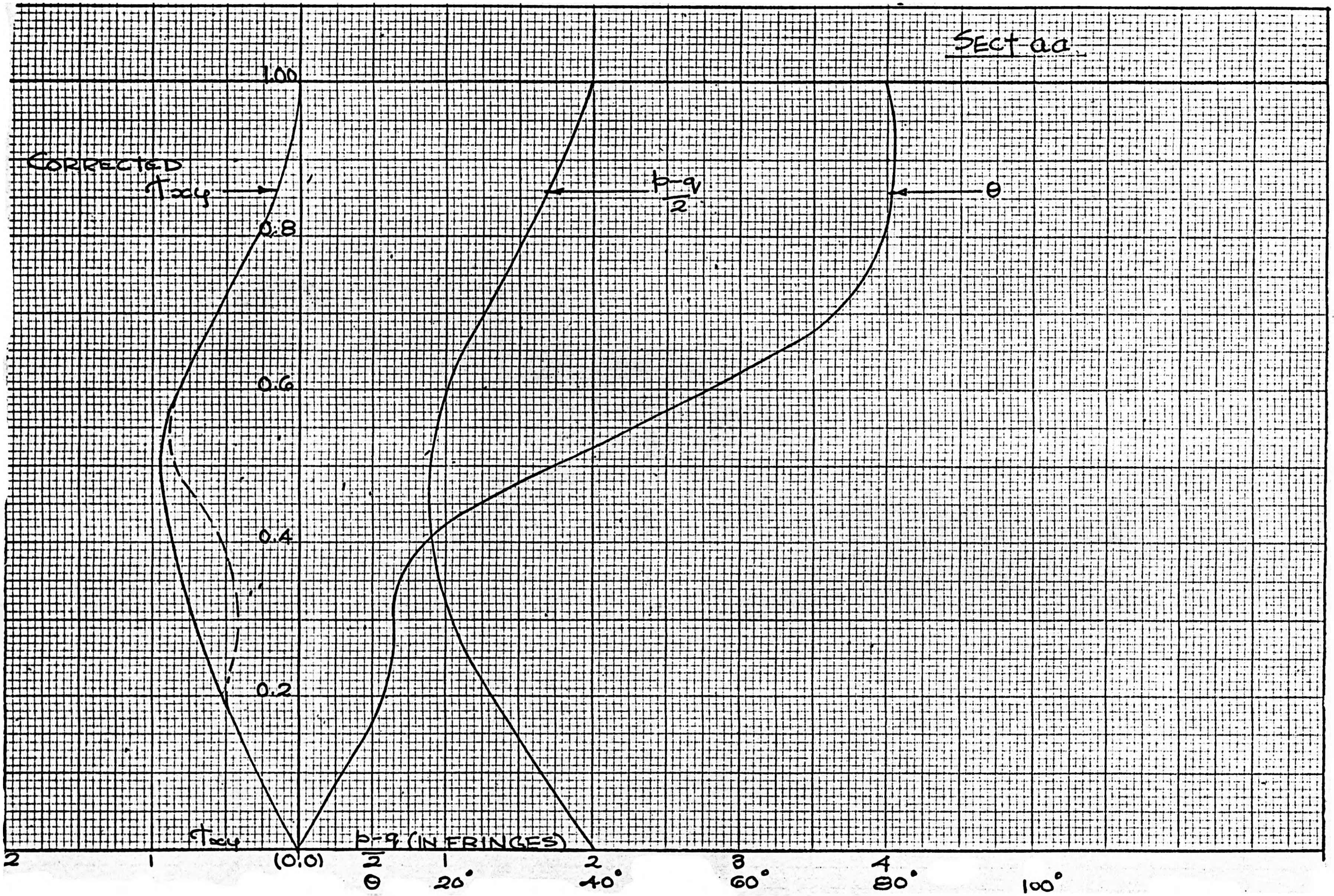


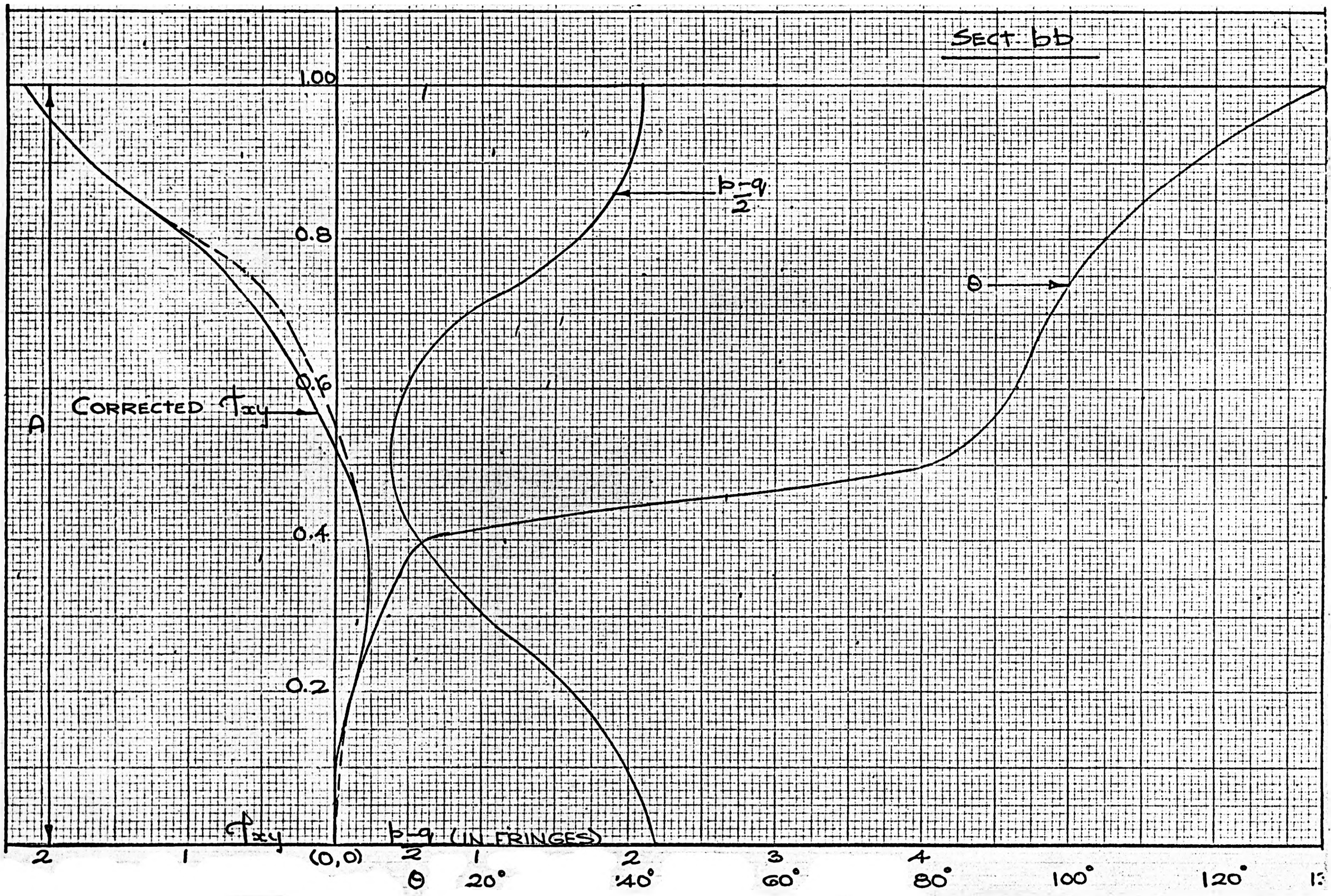
ISOCLINICS

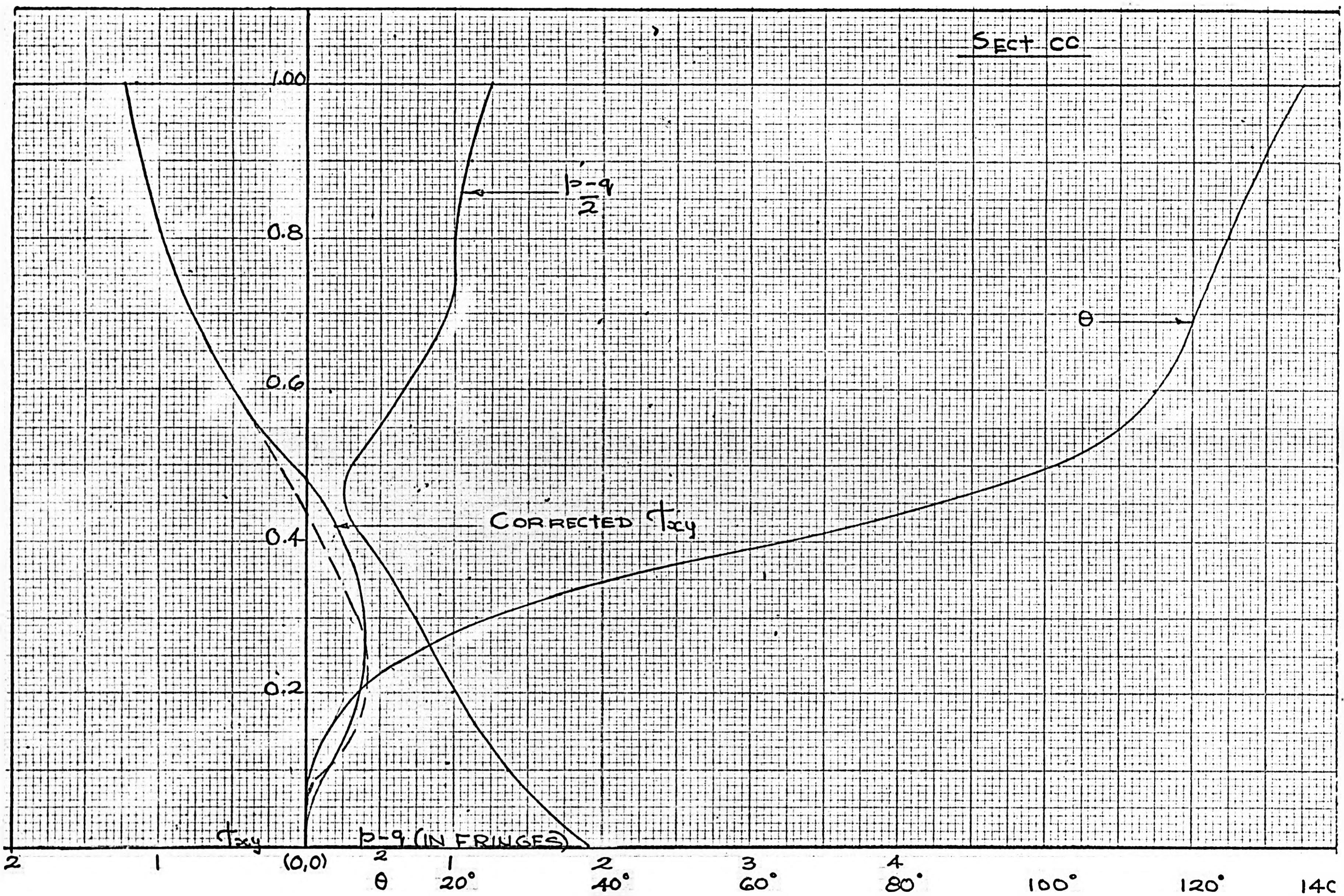


FRINGE PATTERN

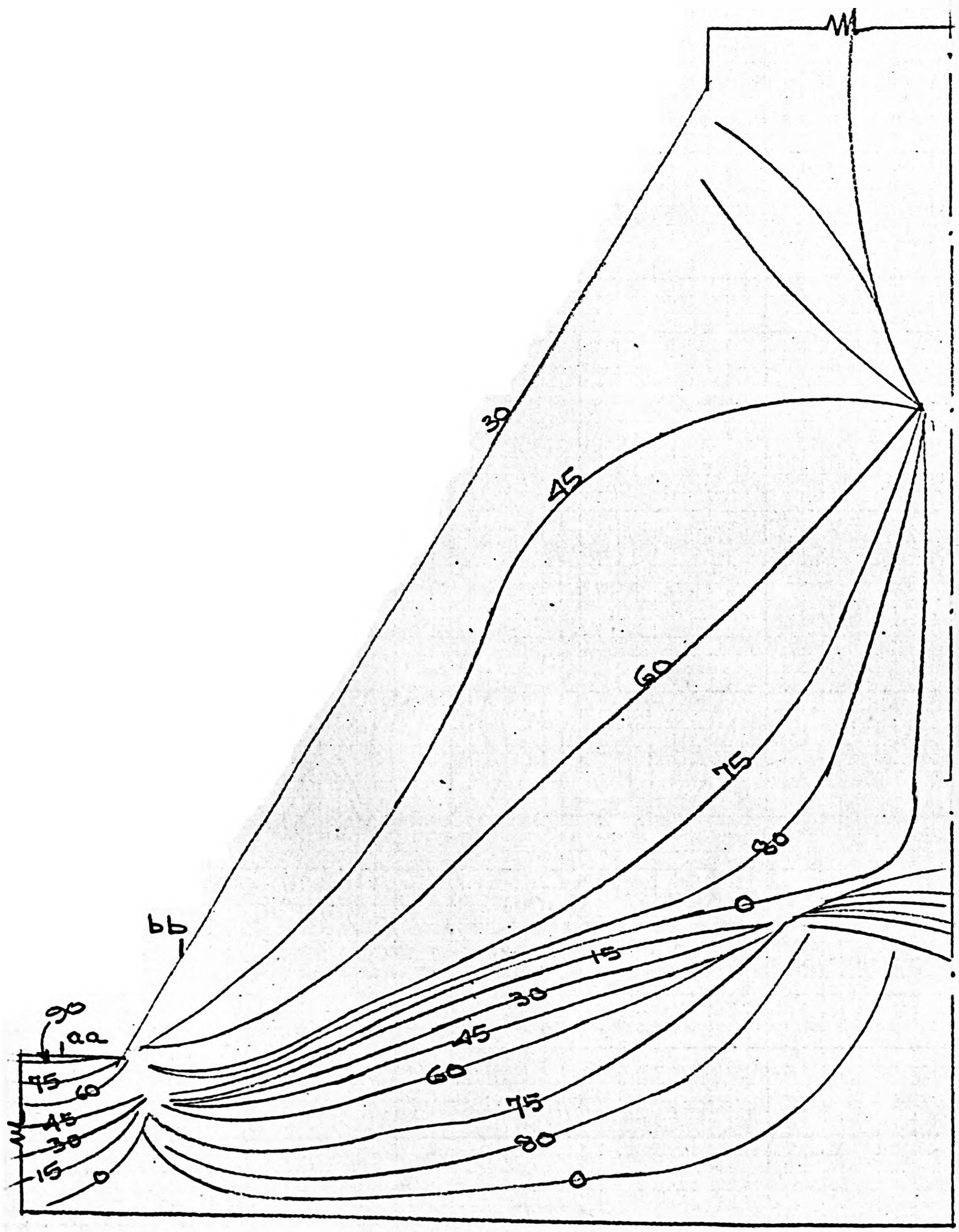
FIG. 12


SECT aa
CORRECTED
τxy
p-q
2
θ
1.00
0.8
0.6
0.4
0.2
τxy
p-q (IN FRINGES)
2
θ
2
1
(0,0)
1
2
3
4
20°
40°
60°
80°
100°


SECT. bb
1.00
0.8
0.6
0.4
0.2
$\frac{p-q}{2}$
θ
A
CORRECTED τ_{xy}
τ_{xy}
$\frac{p-q}{2}$ (IN FRINGES)
2
1
(0,0)
1
2
3
4
θ
20°
40°
60°
80°
100°
120°

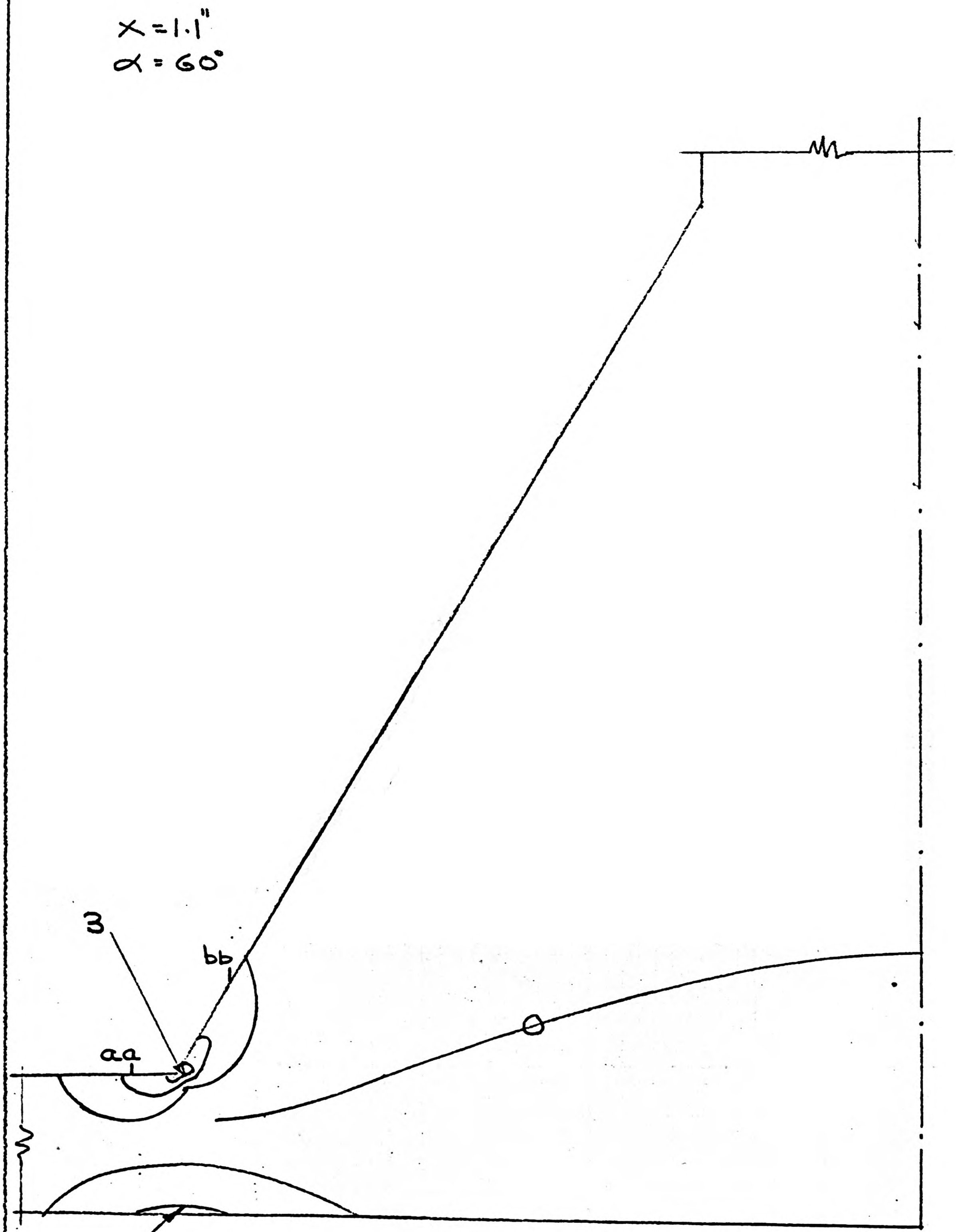

SECT CC
1.00
0.8
0.6
0.4
0.2
p-q/2
θ
CORRECTED τxy
τxy
(0,0)
p-q (IN FRINGES)
2
1
2
1
2
3
4
θ
20°
40°
60°
80°
100°
120°
140

$X = 1.1''$
$\alpha = 60°$



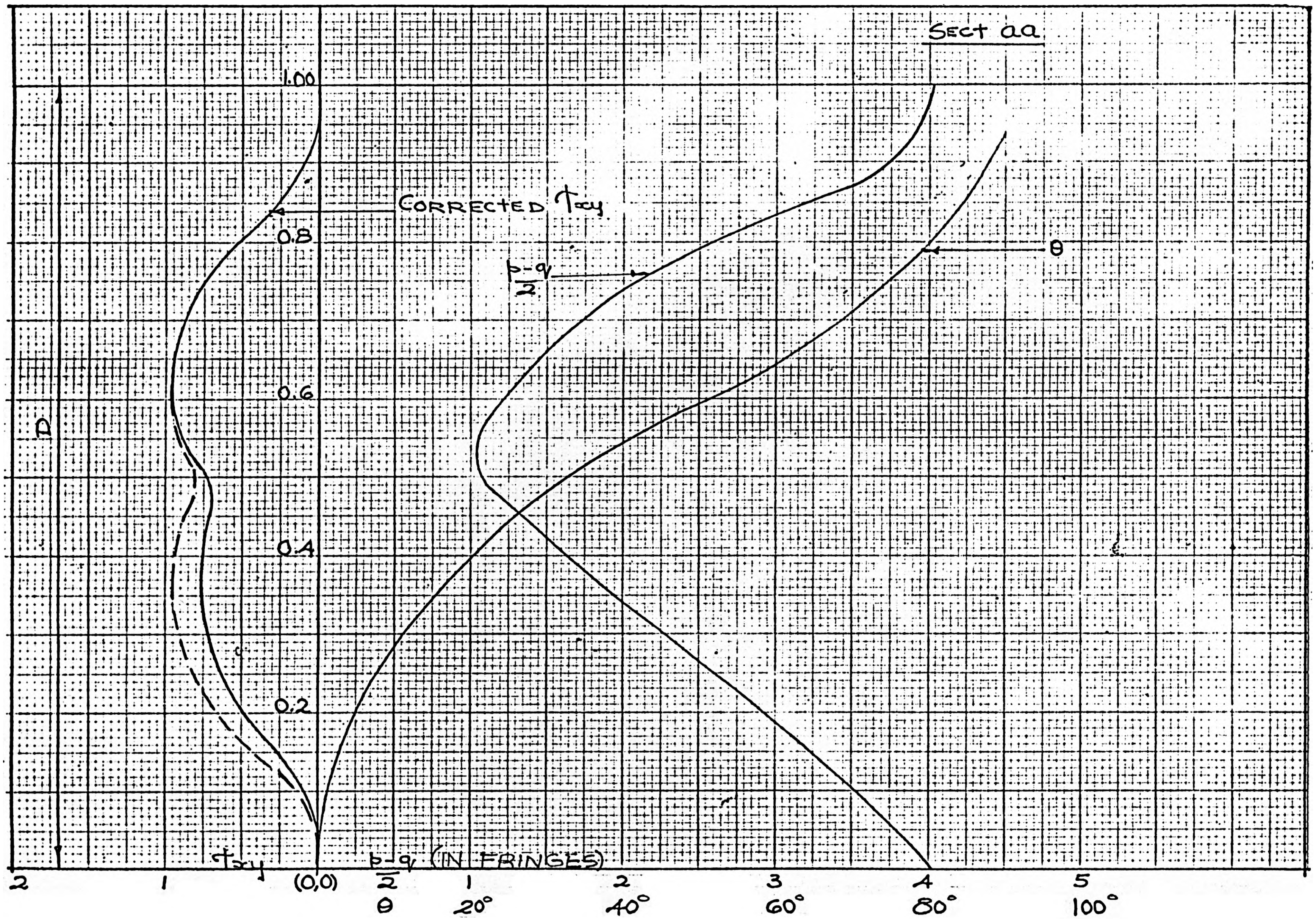
ISOCLINICS

FIG. 12

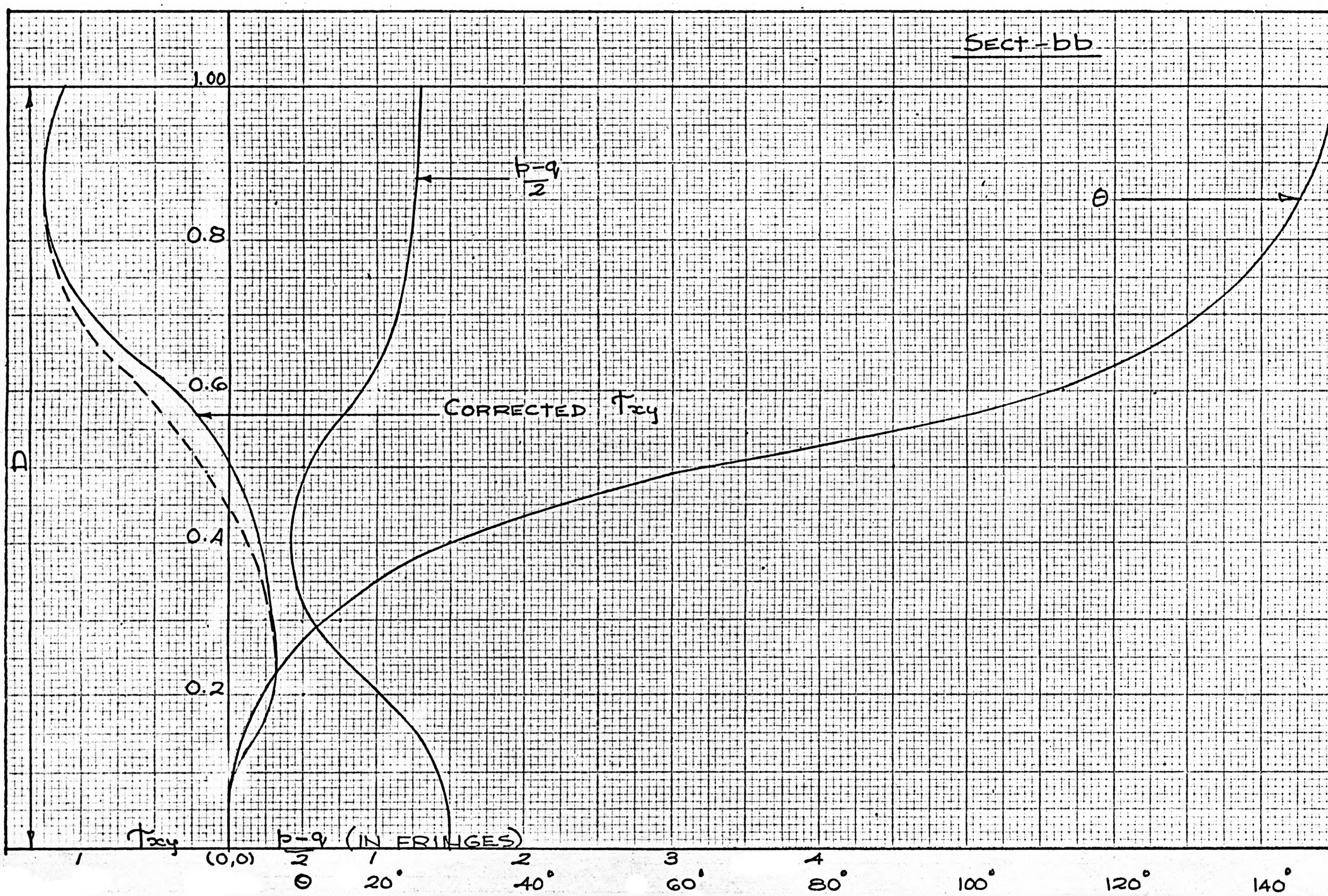


FRINGE PATTERN

FIG.



SECT aa
1.00
0.8
0.6
0.4
0.2
D
CORRECTED τ_{xy}
$\frac{p-q}{2}$
θ
τ_{xy}
(0,0)
$\frac{p-q}{2}$ (IN FRINGES)
2
1
1
2
3
4
5
θ
20°
40°
60°
80°
100°


Sect - bb
1.00
0.8
0.6
0.4
0.2
p-q / 2
θ
Corrected τxy
D
τxy
(0,0)
p-q / 2 (In Fringes)
1
1
2
3
4
θ
20°
40°
60°
80°
100°
120°
140°

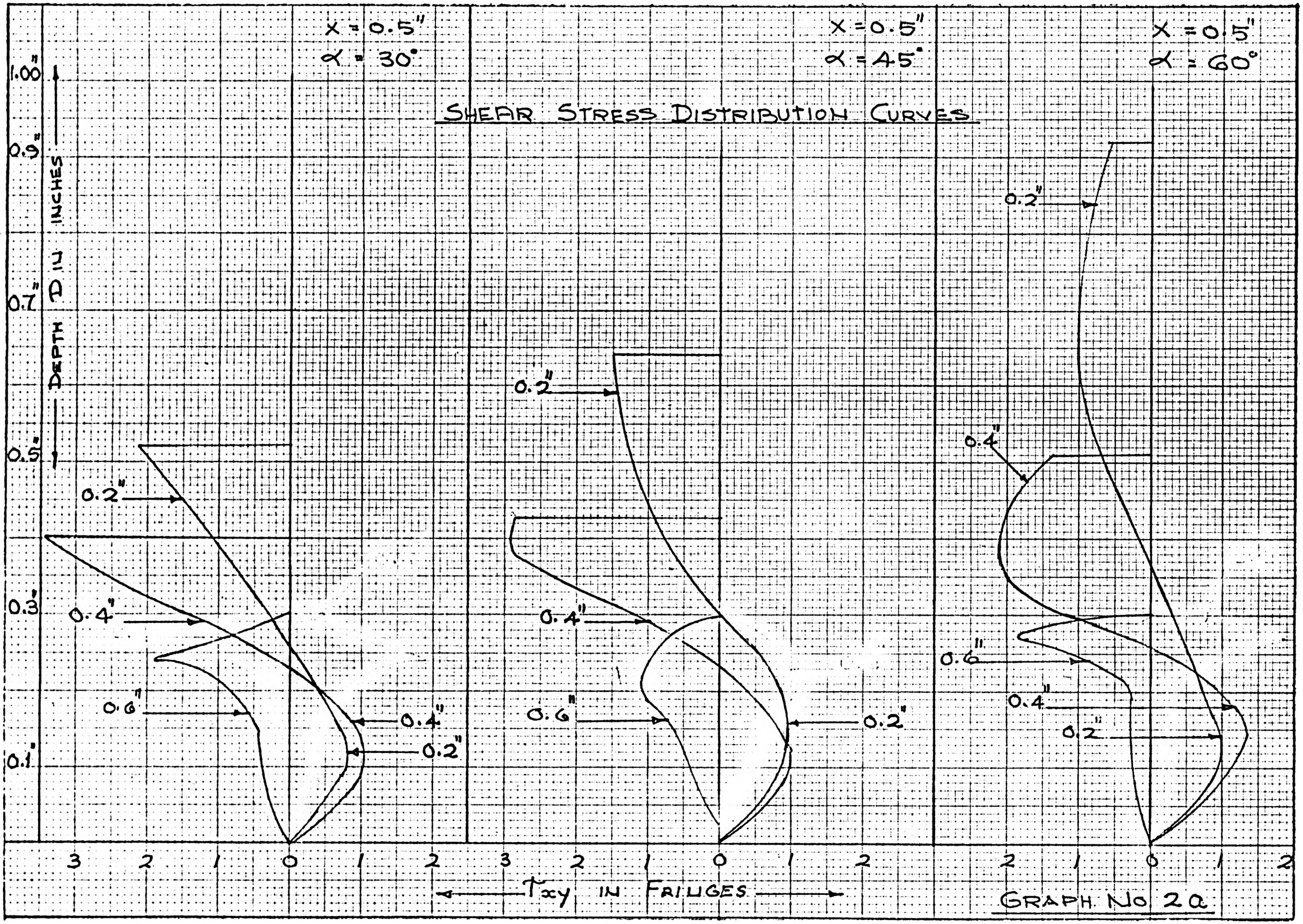
VI. RESULTS

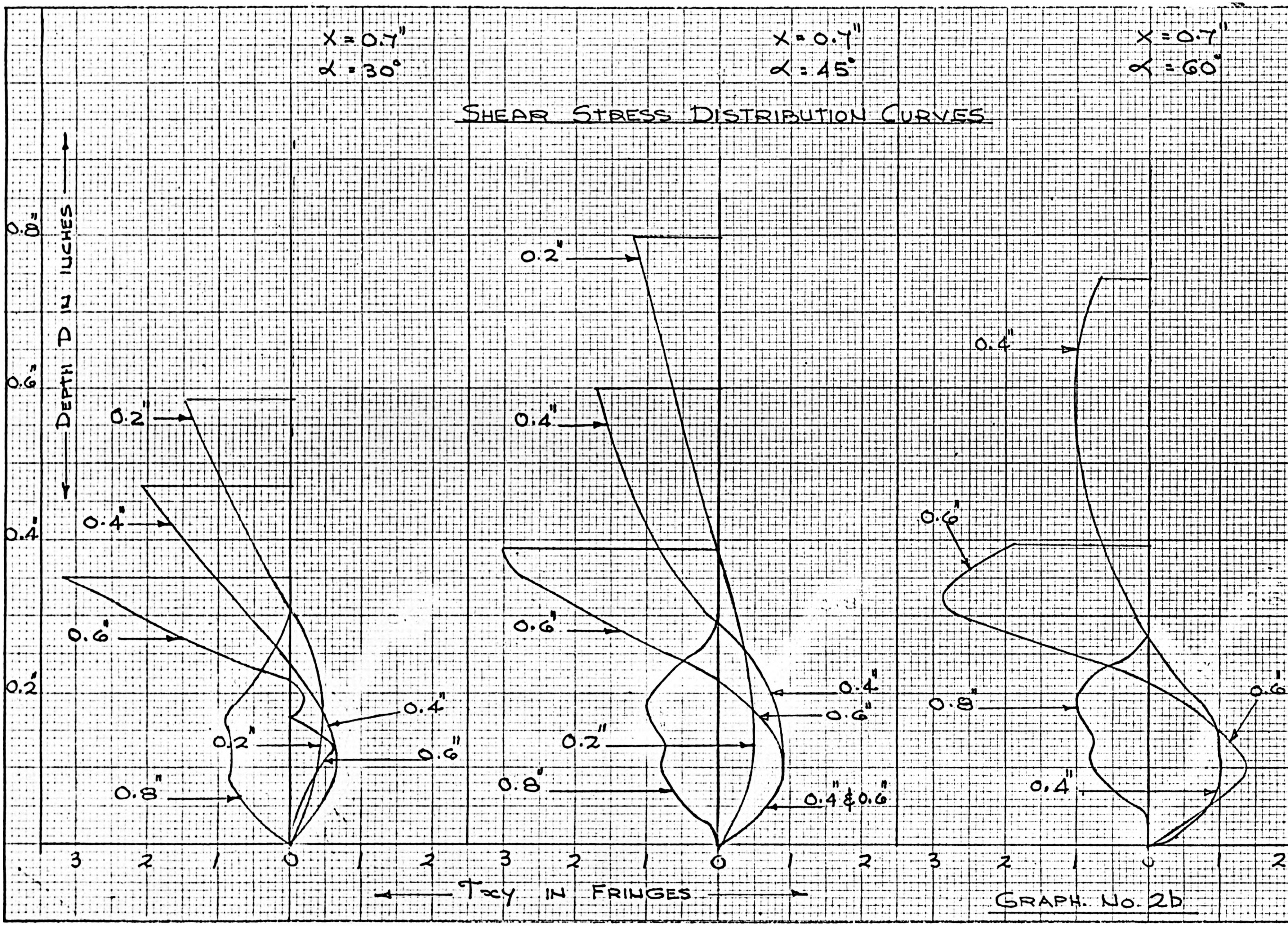
The results of this study are tabulated in table 1. Also some of the results are plotted as shown in graph numbers 3, 4, 5.

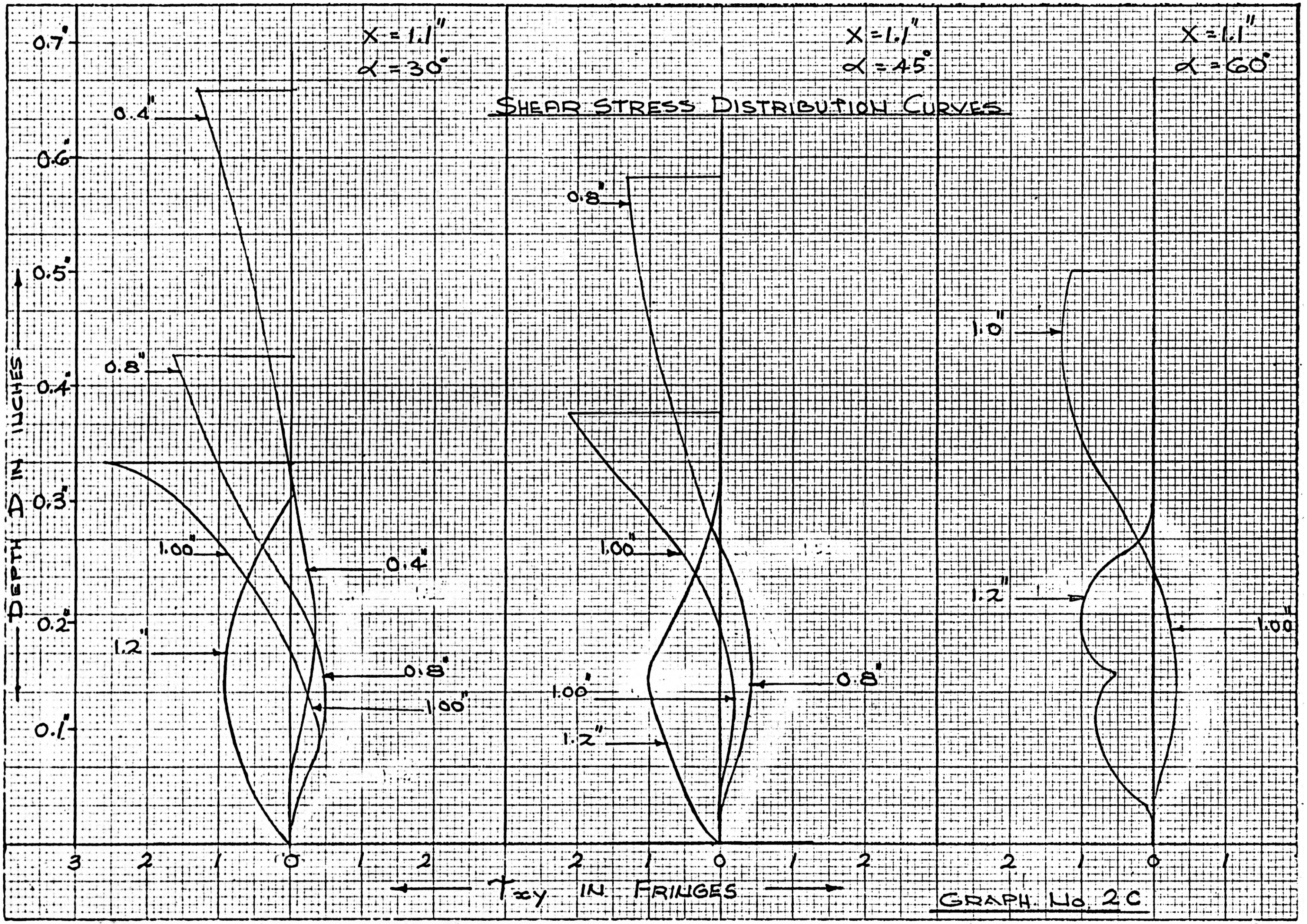
The most obvious effect of the haunch is to decrease shear at the face of the column. Consider a haunch with X = 0.7 inches, and $\alpha = 30°$. At a section, 0.6 inches from the face of the column, the shear stress is 3.2 fringes, but at a section 0.2 inches from the face of the column, the shear stress is 1.75 fringes. Thus as one goes inside the haunch, the shear stress decreases.

If two sections, one on each side of the point where the haunch meets the beam are taken, then the maximum shear stress occurs on the section which is inside the haunch. Both these sections are very close to the above point. For example, if the haunch with X =0.7 inches and $\alpha = 30°$ is considered, then the shear stress on a section at a distance of 0.6 inches is 3.2 fringes while that at a distance of 0.8 inches is 0.85 fringes, and the direction of the shear stress is downwards for both the sections. Thus the critical section lies just inside haunch and not outside as expected due to a sudden decrease in cross-section.

From the graph numbers 2a, 2b, 2c, it is clear that the τ_{xy} curves for 30° attain their maximum value at the inclined face, while those for $\alpha = 45°$ also attain maximum value at the inclined face but the nature of the curve

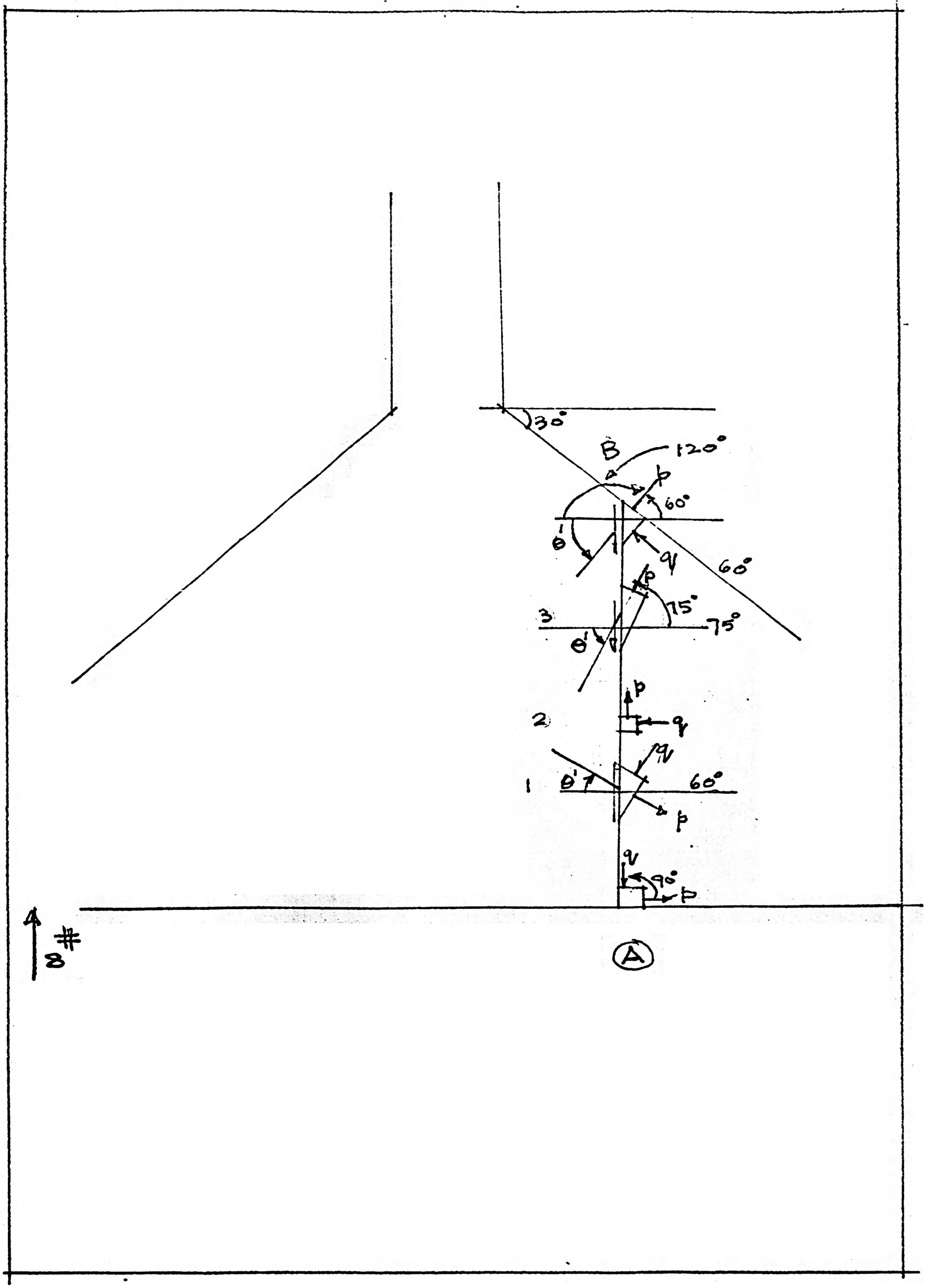

X = 0.5"
α = 30°
X = 0.5"
α = 45°
X = 0.5"
α = 60°
SHEAR STRESS DISTRIBUTION CURVES
1.00"
0.9"
0.7"
0.5"
0.3"
0.1"
DEPTH D IN INCHES
0.2"
0.4"
0.6"
0.4"
0.2"
0.2"
0.4"
0.6"
0.2"
0.2"
0.4"
0.6"
0.4"
0.2"
3 2 1 0 1 2 3 2 1 0 1 2 2 1 0 1 2
τxy IN FRINGES
GRAPH No 2a

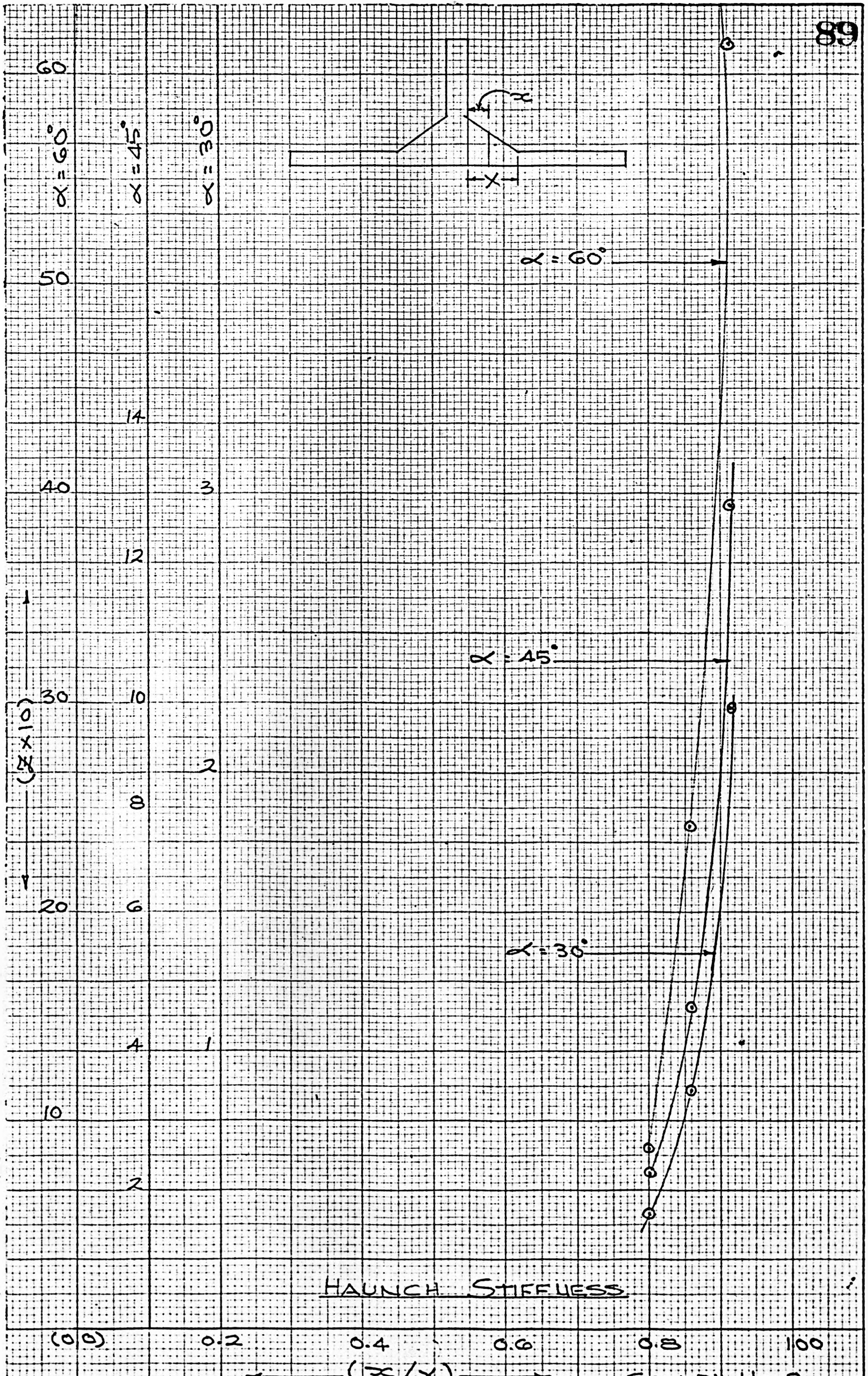

X = 0.7"
α = 30°
X = 0.7"
α = 45°
X = 0.7"
α = 60°
SHEAR STRESS DISTRIBUTION CURVES
DEPTH D IN INCHES
0.8"
0.6"
0.4"
0.2"
0.2"
0.4"
0.6"
0.4"
0.2"
0.6"
0.8"
0.2"
0.4"
0.6"
0.4"
0.6"
0.2"
0.8"
0.4" & 0.6"
0.4"
0.6"
0.8"
0.6"
0.4"
3 2 1 0 1 2 3 2 1 0 1 2 3 2 1 0 1 2
Txy IN FRINGES
GRAPH. NO. 2b


X = 1.1"
α = 30°
X = 1.1"
α = 45°
X = 1.1"
α = 60°
SHEAR STRESS DISTRIBUTION CURVES
DEPTH D IN INCHES
0.7"
0.6"
0.5"
0.4"
0.3"
0.2"
0.1"
0.4"
0.8"
1.00"
1.2"
0.4"
0.8"
1.00"
0.8"
1.00"
1.00"
1.2"
0.8"
1.0"
1.2"
1.00"
3
2
1
0
1
2
2
1
0
1
2
2
1
0
1
τxy IN FRINGES
GRAPH No. 2C

indicates that it is not rising so sharply as 30° curve, and when $\alpha = 60°$, the maximum shear stress occurs somewhere inside the haunch, at 0.8 D as measured from the bottom.

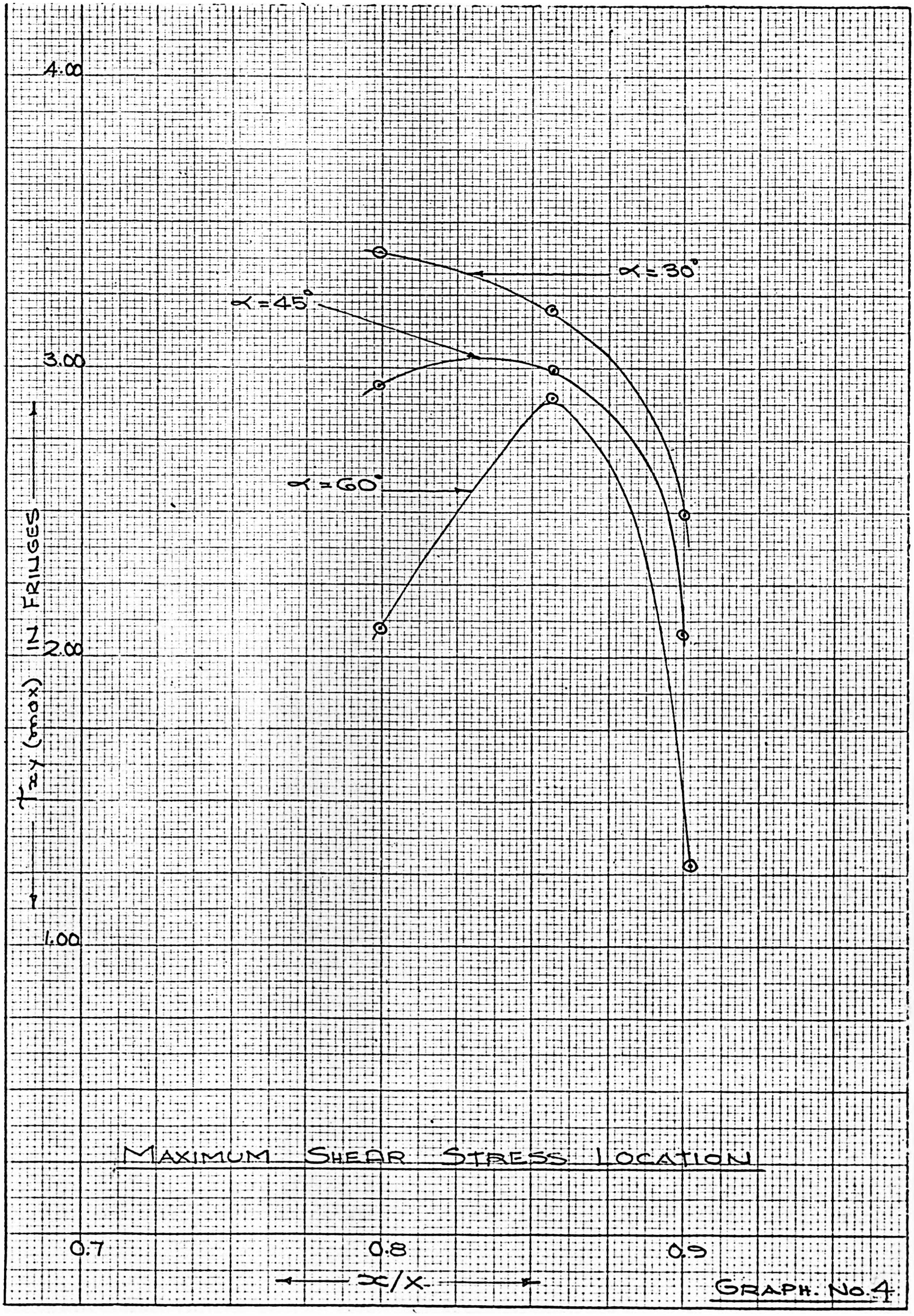
Also reversal of stresses occurs as we enter inside the haunch. In the beam portion, the shear stress is acting downwards as is expected from loading condition, but if a section AB is considered, (fig. 13), then at the bottom of the beam, the stress normal to the boundary is zero, the only stress present is the tensile principal stress parallel to the boundary. This is designated by p. Now, consider a point where a 60° isoclinic cuts the section AB, (1). As p and q are both tensile here, the shear on the face AB of this element must be acting upwards. Similarly (2) represents a state of stress at 0° isoclinic and (3) represents a state of stress for 75° isoclinics. At this point, both the stresses are compressive, but from the equilibrium of the element, it can be shown that the shear at this point must be acting downwards, but for the equilibrium of the free body at AB, the net shear on AB must be acting downwards. This was actually the case, when both areas under τ_{xy} curve were measured by planimeter, and hence it can be safely said that most of the shear is taken up by that material of the haunch which is between the center of the section and the inclined face of the haunch.

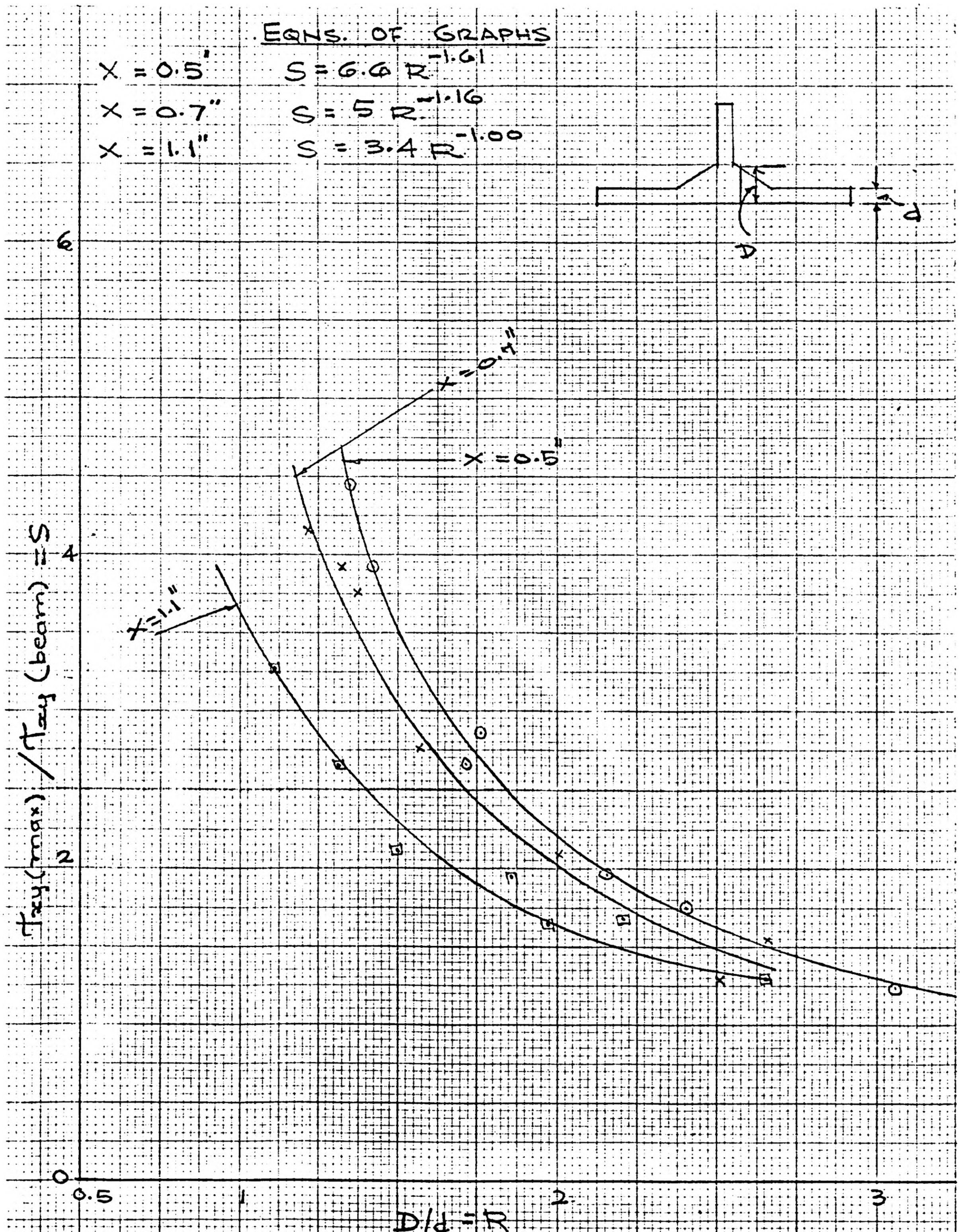

30°
B
120°
p
60°
θ'
q
60°
p
3
75°
θ'
75°
p
2
q
q
1
θ'
60°
p
q
90°
p
8#
A



60
50
40
30
20
10
14
12
10
8
6
4
2
3
2
1
γ = 60°
γ = 45°
γ = 30°
(γ x 10)
x
X
α = 60°
α = 45°
α = 30°
HAUNCH STIFFNESS
(0,0)
0.2
0.4
0.6
0.8
1.00
(x/X)

GRAPH. No. 3


4.00
3.00
2.00
1.00
τ_{xy} (max) IN FRINGES
$\alpha = 30°$
$\alpha = 45°$
$\alpha = 60°$
MAXIMUM SHEAR STRESS LOCATION
0.7
0.8
0.9
x/X
GRAPH. No.4



RELATION BETWEEN DEPTH AND SHEAR

GRAPH No 5

If the graph numbers 3, 4, 5, are examined carefully, then the exact value and the location of the point where maximum shear stress occurs can be found from these three graphs. By knowing the stiffness of the haunch, the ratio x/X for which the shear is maximum can be read from graph no 3 . The maximum value of the shear stress for this particular ratio in fringes can be found from graph no. 4 . Knowing the maximum value of shear stress for a particular haunch and its location, the haunch can be safely designed. This design may not be economical. After designing, the haunch can be checked for any particular section by using graph no. 5 . We know the ratio D/d and then the maximum shear can be read from graph no. 5 . All the above values of shear stresses are in fringes which when multiplied by 207.85 gives the shear stress in psi.

Before closing the discussion on results, it should be mentioned that as in any method of analysis, errors also occur in photoelastic method of analysis. Adjustments in τ_{xy} curves were needed to achieve proper area under them. Sometimes only one trial was needed to adjust the area, sometimes as many as three trials were needed for this adjustment. But in all the cases, maximum efforts were made to get the mean τ_{xy} curve. As can be seen from table 1 the areas under τ_{xy} curves are within 10% of total applied shear.

TABLE I

TABULATION OF RESULTS

X		x	τ_{xy}		y		%Dev.*
inches		inches	Fringes		inches		
			(+)	(-)	(+)	(-)	
0.5"	30°	0.2	0.900	2.15	0.25D	1.00D	0.00
		0.4	1.00	3.4	0.25D	1.00D	-2.00
		0.6	1.9	---	0.8D	---	+8.1
	45°	0.2	0.95	1.50	0.25	1.00	-3.74
		0.4	1.00	2.95	0.25	0.95	+8.00
		0.6	1.1	---	0.7	---	+1.25
	60°	0.2	1.00	0.95	0.13	0.75	+2.5
		0.4	1.35	2.1	0.27	0.8	-5.6
		0.6	1.90	---	0.9	---	-8.4
0.7	30°	0.2	0.45	1.75	0.28	1.00	0.00
		0.4	0.65	2.1	0.25	1.00	+4.45
		0.6	0.6	3.2	0.40	1.00	-4.37
		0.8	0.85	---	0.3 0.55	---	+6.25
	45°	0.2	0.5	1.20	0.20	1.00	+9.37
		0.4	0.9	1.70	0.25	1.00	+4.36
		0.6	0.9	3.00	0.3	1.00	-4.48
		0.8	1.00	---	0.6	---	-3.00
	60°	0.2	---	---	---	---	---
		0.4	1.05	1.00	0.15	0.75	-6.25
		0.6	1.40	2.90	0.3	0.8	-2.5
		0.8	1.05	---	0.65	---	-2.5

TABLE I (CON'T)

1.1	30°	0.2	0.4	1.00	0.3	1.00	+8.5
		0.4	0.4	1.30	0.3	1.00	+7.5
		0.6	0.3	1.50	0.3	1.00	+6.2
		0.8	0.5	1.70	0.3	1.00	+6.25
		1.00	0.4	2.50	0.3	1.00	+4.32
		1.20	1.00	---	0.5	---	-5.00
	45°	<u>0.8</u>	0.4	1.25	0.25	1.00	-2.00
		<u>1.00</u>	0.25	2.10	0.35	1.00	+5.00
		<u>1.20</u>	0.90	---	0.5	---	+3.75
	60°	0.8	---	---	---	---	---
		1.00	0.3	1.25	0.25	0.85	-2.3
		1.20	1.00	---	0.7	---	-2.5

* % Dev. - Difference between the actual applied shear and the shear stress as obtained experimentally.

VII CONCLUSION

From the results as discussed before, the τ_{xy} curves and the graphs as plotted from the results, the following conclusions are drawn.

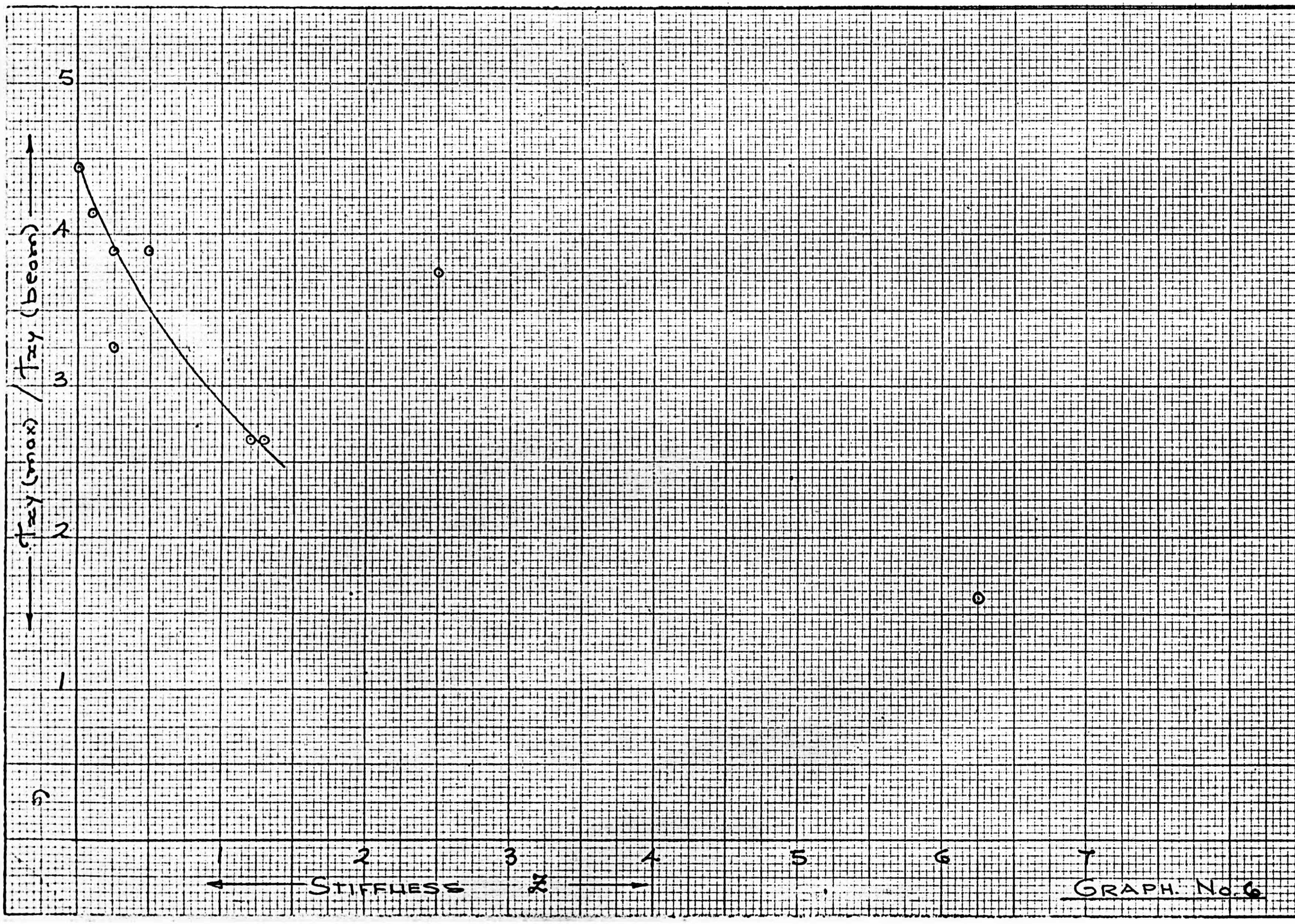
(1) The haunch does relieve the stress concentration at the face of the column but increases the shear stress at the point where the haunch meets the beam.

(2) The maximum shear stress occurs inside the haunch very near to the point of intersection of the haunch and the beam.

(3) The direction of the shear stress is downwards in the beam portion, but inside the haunch the shear stress in the bottom half is upwards while in the top half it is downwards. And downward shear is greater than upward shear for any one section of the haunch. Fig. 13 .

(4) The maximum shear stress for any section of the haunch occurs in the upper quarter of the haunch, near to the inclined face.

(5) For a value of stiffness greater than 1.5 the relationship between stiffness and maximum shear becomes vague and no definite relationship can be expected between them. GRAPH. No. 6.


τxy (max) / τxy (beam)
5
4
3
2
1
0
1
2
3
4
5
6
7
STIFFNESS Z
GRAPH. No. 6

VIII CLOSURE

The results and conclusions as stated before are directly applicable to the problem studied here.

Before applying results to actual structural member, it should be remembered that they have to be modified according to the shape, size and elastic properties of the actual member and material.

It should be noted that the procedure as indicated for the design of a haunch can be specified exactly if prototype models of the materials are actually tested for the confirmation of the results obtained with photoelasticity. Once it is possible to correlate the results of an actual test with those from photoelasticity, it is possible to carry out the same tests for other haunches by varying the depth and breadth of the haunches and thus a specific design procedure can be established.

This is however a completely independent problem and requires an independent treatment. This writer recommends that further study be made in this area.

BIBLIOGRAPHY

1. CROSS, HARDY and MORGAN, N. D., "Continuous Frames

2. PORTLAND CEMENT ASSOCIATION, "A Hand Book of Frame Constants."

3. LYSE, INGE And BLACK, W. E., "An Investigation of Steel Rigid Frames", prosc. A.S.C.E., Nov. 1940.

4. OLANDER, HARVEY C., "A Method for Calculating Stresses in Rigid Frame Corners", prosc. A.S.C.E., August, 1953.

5. TIMOSHENKO, S., "Strength of Materials", Part II

6. HEAGLER, RICHARD B., "Gathering Haunch Design Data by Photoelasticity", a Thesis - Missouri School of Mines and Metallurgy, 1962.

7. FOGEL, CHARLES M., "Introduction to Engineering Computations."

8. DURELLI, A. J.; PHILLIPS, F. A.; and TSAO, C. H., "Analysis of Stress and Strain."

9. FROEHT, M. M., "Photoelasticity", Vol. I.

10. HEYWOOD, R. B., "Designing by Photoelasticity."

VITA

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